PRNC - 110

PUERTO RICO NUCLEAR CENTER

PLOWSHARE WORKSHOP

July 1 through August 15, 1967

Conducted by

J. A. Cheney and W. K. Talley

University of California, Davis

Offered by the

Division of Nuclear Engineering

Donald S. Sasscer, Head

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OPERATED BY UNIVERSITY OF PUERTO RICO UNDER CONTRACT NO. AT (40-1)-1833 FOR U.S. ATOMIC ENERGY COMMISSION

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INTRODUCTION

In 1964, the third Plowshare* symposium was held on the Davis Campus of the University of California under the sponsorship of the Lawrence Radiation Laboratory at Livermore (LRL), the Atomic Energy Commission, the American Nuclear Society, the American Society for Engineering Education (ASEE), and the Department of Applied Science, U. C. Davis. An ASEE committee used the papers presented at the symposium to assess the suitability of introducing Plowshare technology into academic curricula. The committee[(1)] felt that such action was premature. Their contention was that the field was still in the research and development stage. Further, they felt that the science and technology of nuclear explosives was not sufficiently codified for instruction, and that the expertise to do so was available only in a few laboratories, such as LRL and Sandia.

Publication of the symposium papers and a growing awareness of the potentials of the nuclear explosive as an engineering tool have modified the situation. Since 1964, several schools have added descriptions of the effects and uses of underground nuclear explosives to regular engineering courses or have started formal instruction in Nuclear Civil Engineering.[(2,3,4)] Two of these innovations are within 150 miles of

*The program to make constructive use of nuclear explosives is known as the Plowshare Project.

(1) E. J. Goldberg, "The Third Plowshare Symposium," J. Eng. Educ., 55, 4, 100 (1964).

(2) P. Kruger, "A Graduate Level Course in Nuclear Civil Engineering," Trans. Am. Nuc. Soc., 9, 1, 208 (1966).

(3) W. K. Talley, "Plowshare in University Programs," Trans. Am. Nuc. Soc., 9, 1, 312 (1966).

(4) P. Kruger, "Nuclear Explosives as an Engineering Subject," Nuclear News, 10, 4, 18-20 (1967).

each other: at Stanford University and at the University of California at Davis. The Stanford course is a 30 quarter hour lecture series taught by Prof. Paul Kruger of the Civil Engineering Department. (The content of this course is briefly described in reference 2 and is available in the form of a syllabus -- see Table 1.) At Davis, the phenomenology and physics of nuclear explosions is given in 30 quarter hours in the Department of Applied Science. The uses of these compact energy sources is given in a subsequent course (also 30 quarter hours) taught in the Civil Engineering Department. (The content of these courses include a good deal of the topics presented in Table 2. The main text for both courses is The Constructive Uses of Nuclear Explosives -- see Table 1.) Both Stanford and Davis restrict their courses to graduate students.

It has been slightly more than a decade since the Plowshare program was started at LRL. Large-scale excavations, such as a new sea-level transisthmian canal, have been studied carefully as to their practicality. The personnel at LRL, in cooperation with other researchers, have performed many of the experiments needed to provide data for engineering design. A Presidential Commission is currently studying possible routes and construction methods for a sea-level transisthmian canal. Two routes proposed would use nuclear explosives.

While such spectacular excavating projects tend to attract public attention, there may be more potential in the development of the technology of completely contained underground explosives. Such an explosion must be designed to reduce structural damage due to ground motion, but the hazard of radioactivity release can be eliminated. Project Gasbuggy, scheduled for fall, 1967, will test the ability of nuclear explosives to stimulate natural gas production. Other experiments, planned or proposed, will seek to add to the exploitable resources of the world: oil, oil shale, minerals, water.

SUMMER WORKSHOP PROGRAM AT UPR/PRNC

In addition to acting as a center for Latin American research in nuclear science and engineering, the Puerto Rico Nuclear Center (PRNC) is charged by the United States Atomic Energy Commission with the dissemination of nuclear technology throughout Latin America. This includes instruction in the nuclear sciences and engineering. The dual purposes of the PRNC join naturally in the transisthmian canal: when the canal is built, Latin American engineers will play a large role and, if nuclear explosives are used, there must be available a local training center well versed in Plowshare technology. The Nuclear Center and the Engineering Faculty of the University of Puerto Rico wished to broaden their role in nuclear engineering and science and at the same time prepare the University of Puerto Rico to be a Plowshare training center.

Dr. Henry J. Gomberg, Director of the PRNC, invited Profs. J. A. Cheney and W. K. Talley to lead a three-month workshop at the Mayaguez branch of PRNC. These two people were responsible for the development of the program in Nuclear Civil Engineering at the University of California at Davis. The purpose of the workshop was to provide a complete background for members of the scientific staff of the PRNC and for the faculty members of the UPR -- the scientific bases, the phenomenology, the engineering principles, and the constructive uses of nuclear explosives -- and to then let the UPR design a course for its own students.

The effort was interdisciplinary; there were representatives from the departments of chemical, civil, mechanical and nuclear engineering, physics, and staff members from PRNC. Despite the fact that most of the projected uses fall into the traditional province of the civil engineer (excavation, mining, petroleum reservoir stimulation, aggregate production, water resource development and conservation, etc.), all other engineering fields are touched by the technology of nuclear explosives. This is because the nuclear explosive is not simply a scaled-up conventional high explosive, but introduces its own peculiarities, e.g., energy densities in the megabar range, shocks strong enough to vaporize rock, large earth motion due to spalling, neutron induced radioactivity, fission product radioactivity, etc.

LECTURES IN NUCLEAR CIVIL ENGINEERING

The choice of material covered during the workshop was governed by the familiarity of J. A. Cheney and W. K. Talley with their courses at Davis. It was recognized that the participants were well grounded in one or more of the topics presented (e.g., basic nuclear science, or hydrodynamics, or construction practice, or structural engineering), but no one person was familiar with them all. Table 3 presents the hour by hour lecture topics given to provide an introduction into all the areas covered by nuclear civil engineering; perhaps more importantly, the program provided the faculty group with a common vocabulary. The source material was either from the text by Teller, Talley, Johnson and Higgins (No. 1 Table 1) or from notes included in Appendix A of this report.

The notes comprise an introduction into civil engineering to persons working in the Plowshare technology, who have little or no civil engineering background. They serve only as a brushing of the surface in order that all the participants will have a basic understanding of the approach taken in civil engineering to certain basic problems associated with the constructive use of nuclear devices.

The first two lectures were concerned with the problems of organization and management of a large construction project. This was followed by twelve lectures on engineering mechanics and soil mechanics as applied to problems associated with the design of nuclear civil engineering projects. The topics include the theory of elasticity, elastic waves, failure theories and mechanics, the stability of slopes, ground water flow, the transient response of saturated soil, the dynamic analysis of structures subject to ground motion or air blast.

The last two lectures included in this set of notes deal with the drilling costs for large diameter holes in soil or rock and the problems of numerical instability associated with computer solutions.

As each topic was presented, it was contrasted and compared with the same topic as it would have been presented in a "traditional" engineering discipline. Two examples may make this clear:

In developing the Rankine-Hugoniot equations for shocks, a compressible fluid was used for an analogy. When use was made of the resultant equations to discuss phenomenology, it became clear that a hydrodynamic shock traveling through rock is not entirely analogous to one in a fluid. In granite, as the stress level of the shock declines, there is formed a sonic, elastic 40-kilobar wave that precedes a slower, several hundred kilobar plastic wave. At stress levels above 300 kb and at those below the elastic limit, there is but one compression wave.

In the discussion of engineering practice it was pointed out that on construction projects contractors would be likely to submit fixed price bids, if conventional methods are to be used. On the other hand, if nuclear explosives are specified, the bid is likely to be cost plus fee. Hence, two bids for doing the same job should not be compared directly when one is based on using nuclear explosives and the other is based on the use of outline for the content of a graduate course in nuclear civil engineering to be taught at UPR in the Spring of 1968.

The notes in Appendix A and the text "Constructive Uses of Nuclear Explosives" by Teller, Talley, Higgins and Johnson comprise the material presented in satisfaction of the first item above.

A one semester (40 contact hours) course is suggested for the spring semester 1968 to be offered in cooperation between the Departments of Nuclear and Civil Engineering. It should be a graduate level course (600 series), carry three units credit, and consist of the topics listed in Table 2. In order to fit in the time limit of one semester the topics of Thomas-Fermi Model, Hydro Codes, Equations of Hydro Dynamics, Planning and Organization of Construction Projects and certain scientific applications had to be omitted. However, it was the consensus of the discussion near the end of the workshop that a treatment of similitude and modeling should be added.

The expanded outline of this course as suggested by Messrs. N. Beylerian J. A. Cheney, K. Pedersen, and W. K. Talley is given in Table 4. Also, in order to facilitate the instruction of the course, a set of typical home work problems have been compiled and listed in Appendix C.

To accompany this instructional program and to provide thesis topics for their graduate students, the faculty are widening their research interests to include problems in nuclear civil engineering. For example, they have been conducting research on the solubility of certain copper ores to determine the feasibility of mining large, low-grade ore bodies in in-situ leaching. They are now planning to subject ore samples to transient shocks in the 10 to 100 kb range and will determine what effect, if any, this will have on solubility. These stress levels can easily be produced in an ore body by the use of nuclear explosives.

The crossing of traditional departmental boundaries, often required in research such as this, is perhaps an additional advantage to the University, of programs in nuclear civil engineering.

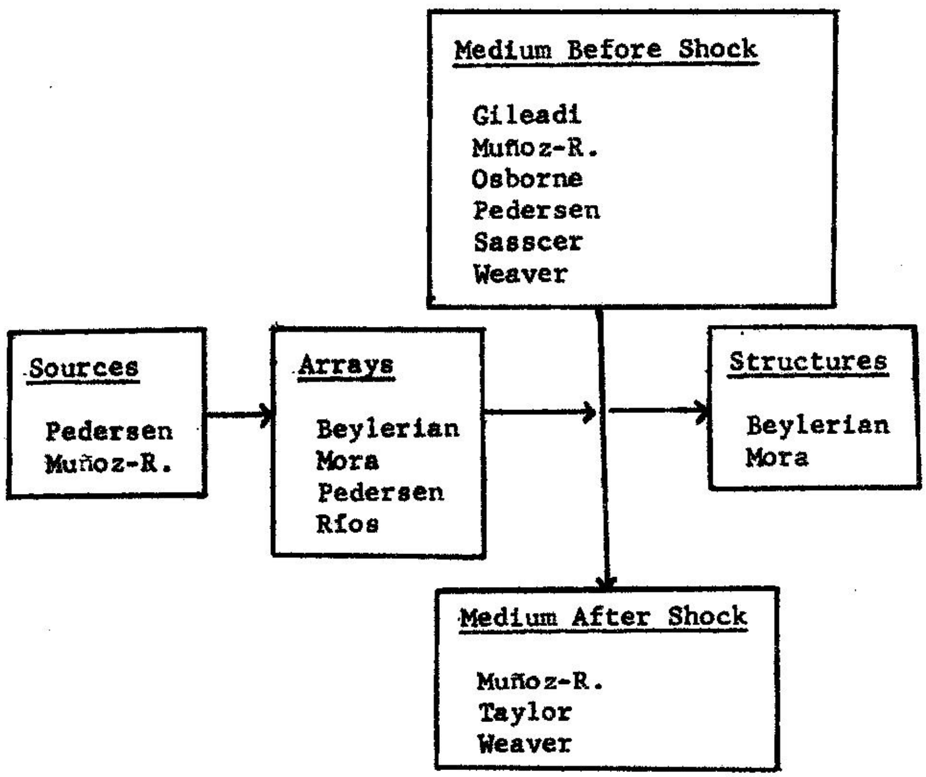


Figure 1

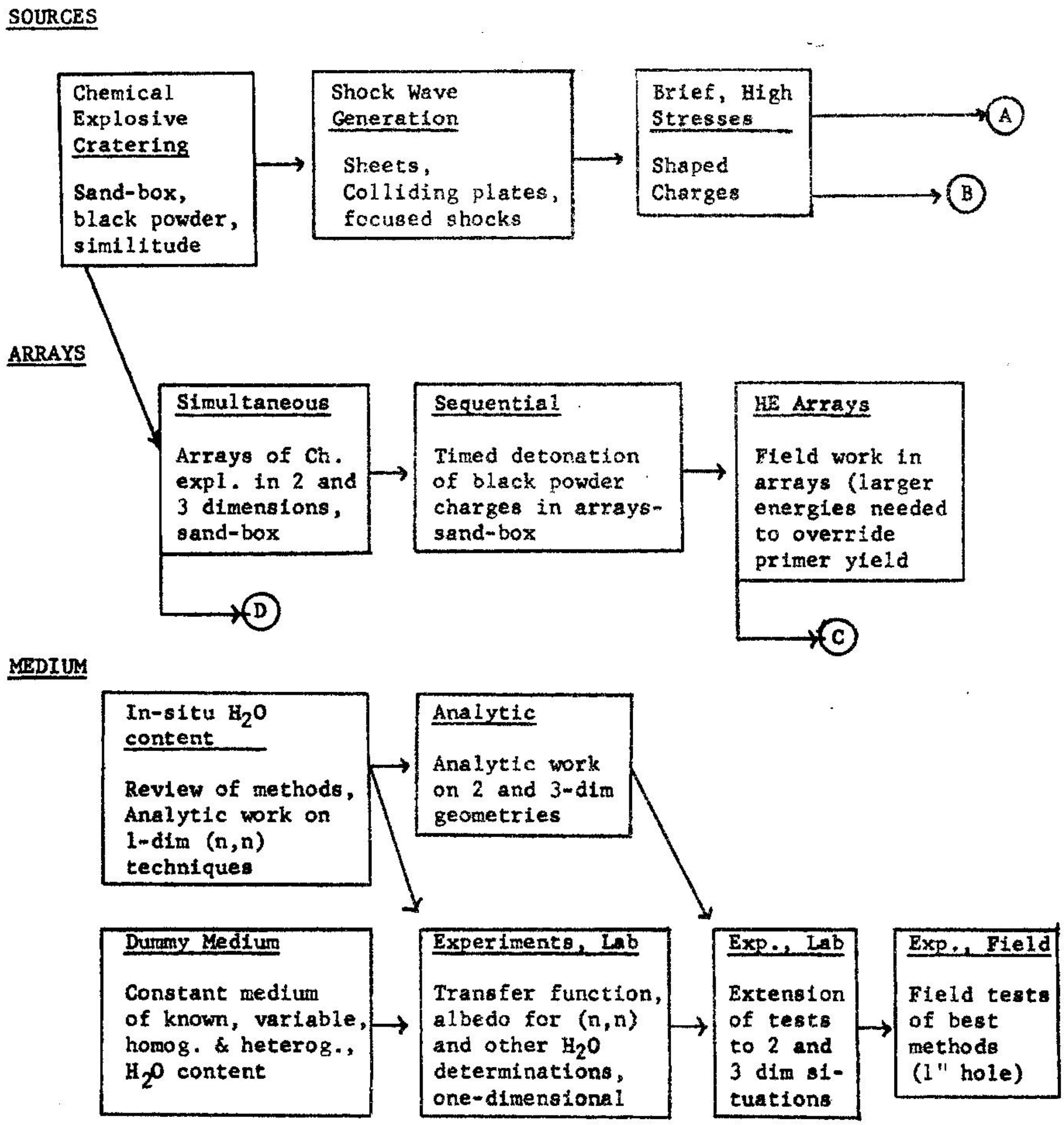
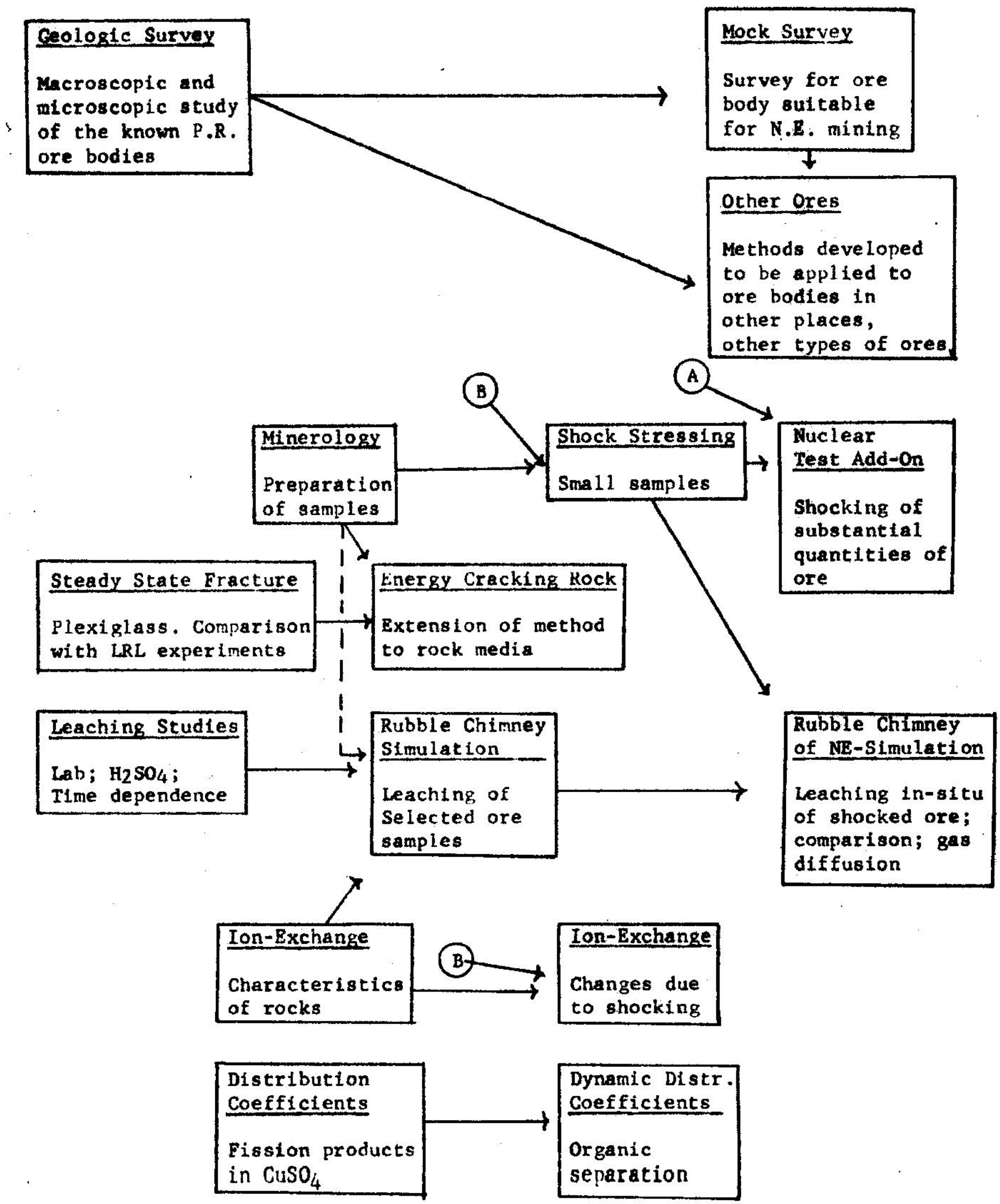


Figure 2. Project Network


Geologic Survey
Macroscopic and microscopic study of the known P.R. ore bodies
Mock Survey
Survey for ore body suitable for N.E. mining
Other Ores
Methods developed to be applied to ore bodies in other places, other types of ores
A
B
Minerology
Preparation of samples
Shock Stressing
Small samples
Nuclear Test Add-On
Shocking of substantial quantities of ore
Steady State Fracture
Plexiglass. Comparison with LRL experiments
Energy Cracking Rock
Extension of method to rock media
Leaching Studies
Lab; H_2SO_4; Time dependence
Rubble Chimney Simulation
Leaching of Selected ore samples
Rubble Chimney of NE-Simulation
Leaching in-situ of shocked ore; comparison; gas diffusion
Ion-Exchange
Characteristics of rocks
B
Ion-Exchange
Changes due to shocking
Distribution Coefficients
Fission products in $CuSO_4$
Dynamic Distr. Coefficients
Organic separation

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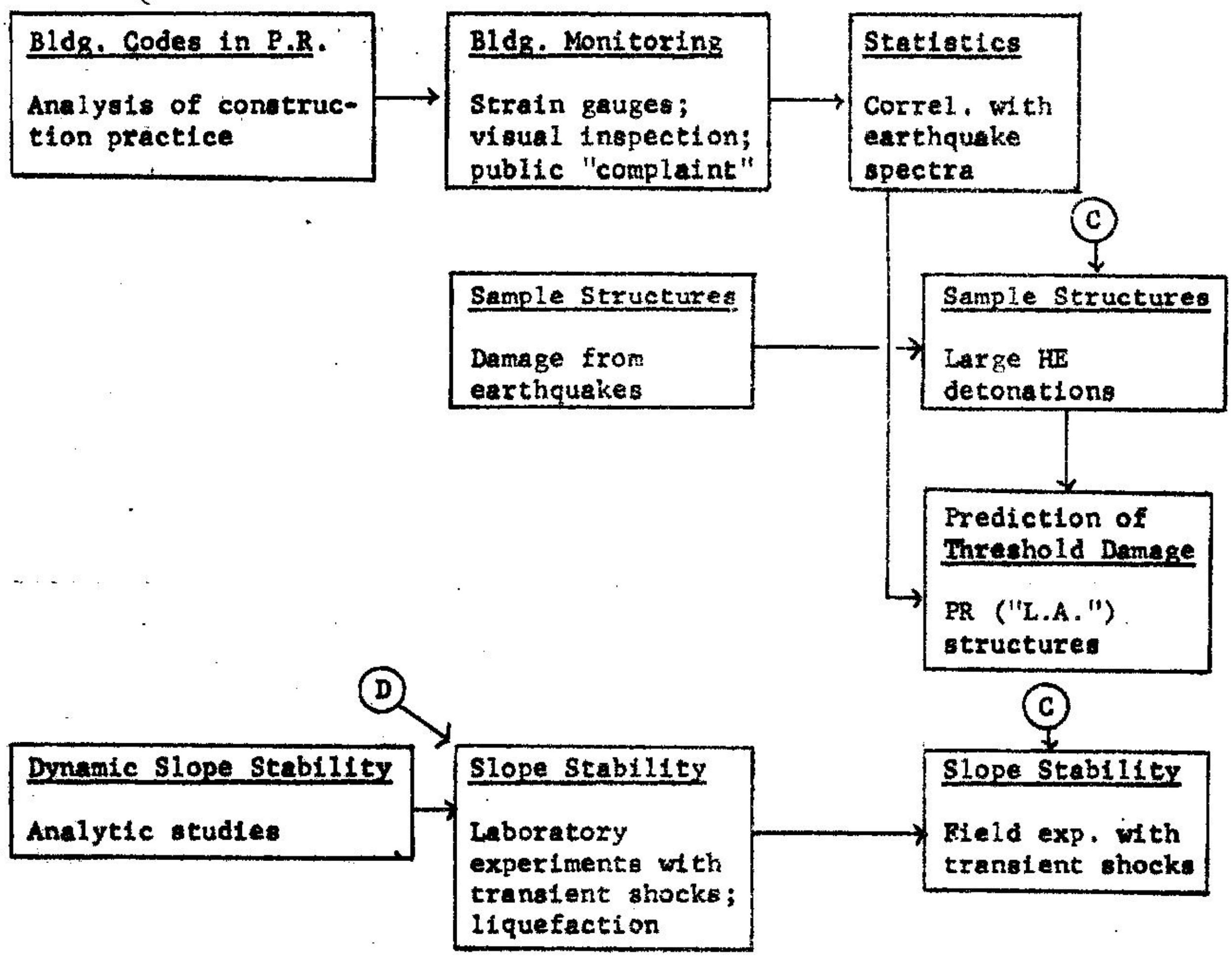
STRUCTURES



A general treatment of the Plowshare Project, including a lucid development of the political-social implications of successful employment of nuclear explosives, is available in:

7. "Project Plowshare, the Development of the Peaceful Uses of Nuclear Explosives," R. Sanders, Public Affairs Press, Washington, D. C., (1962)

The following basic texts have served as useful references for the non-civil engineer studying Nuclear Civil Engineering:

8. "Basic Soil Engineering," B. K. Hough, Ronald Press, New York, 1965.

9. "Project Management with CPM and PERT," J. J. Moder and C. R. Phillips, Reinhold, New York, 1964.

10. "Construction Planning, Equipment, and Methods," R. L. Peurifoy, McGraw-Hill Book Co., New York, 1956.

11. "Handbook of Engineering Mechanics," W. Flugge, McGraw-Hill Book Co., New York, 1962.

TABLE 2 (Cont'd)

Engineering Applications	Scientific Applications
Earthmoving Applications:	Neutron physics
Canals and mountain cuts	Nuclear structure
Harbors	Seismology
Water resource development	Meteorology
Contained Applications:	Chemistry
Aggregate production	Material science
Petroleum reservoir stimulation	
Underground storage	
Tar sands and oil shale	
Mining	

TABLE 3. SEQUENCE OF LECTURES IN PLOWSHARE WORKSHOP - PUERTO RICO - 1967

SOURCES: 1 CUNE
2 Notes
3 Participant's lectures

Lect. No.	Source	Topic
1	1	Brief History of Plowshare Project and Comparison of Energy Sources
2	2	Organization of Civil Engineering Construction Project
3	1	Nuclear Radiation and Radiation Hazards
4	2	Basic Equations of Hydrodynamics
5	2	The Critical Path Method of Scheduling and Project Control
6	1	Foundations of Statistical Mechanics
7	1	Thermodynamics, Compressibility and the Virial Theorem
8	2	Basic Equations of the Theory of Elasticity
9	1	Rankine-Hugoniot Equations
10	1	Equations of State; the Thomas-Fermi Model
11	2	Waves in Elastic Media (Basic Equations)
12	2	Solutions to the Elastic Wave Equations
13	1	Reflection and Interaction of Shocks
14	1	Radiant Energy Content and Transport
15	2	Failure Theories in Mechanics
16	2	Failure Theory in Soil Mechanics
17	2	Rudimentary Analysis of Slope Stability
18	2	Drilling of Large Diameter Holes in Soil and Rock
19	1	Nuclear Explosives: Size, Shape, Weight, Cost and Yield
20	1	Production of Radioactivity - Fission and Fusion
21	2	Flow in Porous Media
22	2	Seepage Forces

TABLE 3 (Cont'd)

Lect. No.	Source	Topic
23	1	Nuclear Excavation - Fallout
24	1	Transport of Radioactivity in Groundwater
25	2	Settlement in Saturated Soils
26	1	Phenomenology: Contained Explosion
27	1	Phenomenology: Cavity, Chimney, Crater
28		Movie: Project SEDAN
29	2	Structural Dynamics
30	2	Structural Response; Spectral Analysis
31	3	Geological Description of the Island of Puerto Rico
32	2	Structural Response: Seismic
33	1	Phenomenology: Craters
34	1	Measurement of Explosions: Instrumentation
35	1	Prediction of Explosions: Hydrocodes
36	1	Shock Wave Measurements and Predictions
37	2	Mathematical and Physical Instabilities
38	1	Hazards Evaluation - Groundshock and Blast
39	1	Neutron Physics
40	1	Other Engineering Parameters
41	1	Isotope Production, Seismology
42	1	Earthmoving Applications; Water Resource Conservation and Development
43	1	Meteorology; Upper Atmosphere, Space
44	1	Fracturing, Rubble Size Distribution, Chimney Tonage, Aggregate Production
45	1	Tar Sands and Oil Shale
46	3	Transisthmian Canal

TABLE 3 (Cont'd)

Lect. No.	Source	Topic
47	3	Carryall (Highway Cut)
48	3	Chariot (Harbors)
49	3	Gasbuggy (Gas well stimulation)
50	3	Mining
51	3	Geothermal Heat/Simulation
52	3	Neutron Diffusion and Isotope Production

TABLE 4. COURSE SCHEDULE IN NUCLEAR CIVIL ENGINEERING AT UPR

Lect. No.	Topic
1	The Plowshare Project
2	Nuclear structure and energy sources
3	Nuclear radiation and hazards
4	Cost, size, shape, and emplacement
5	Determination of yield and production of radioactivity
6	Distribution of radioactivity: fallout and groundwater
7	Thermodynamics and compressibilities
8	Theory of elasticity
9	Waves in elastic media
10	Rankine-Hugoniot relations
11	Shocks and shock adiabats
12	Spallation due to shocks
13	Interaction and reflection of shocks
14	Contained explosions: cavity, chimney, scaling
15	Cratering explosions: apparent crater, scaling
16	Similitude and modeling
17	Medium properties and instrumentation
18, 19	Structural dynamics
20	Hazards due to groundshock and airblast
21, 22, 23	Failure theories in mechanics (soils)
24	Slope stability

TABLE 4 (Cont'd)

Lect. No.	Topic
25	Flow in porous media, seepage forces
26	Settlement of saturated soils
27, 28	CPM
29, 30	Canals, mountain cuts, and harbors
31	Water resource development
32	Aggregate production
33, 34	Petroleum reservoir stimulation; underground storage
35	Tar sands and oil shale
36, 37	Mining
38	Nuclear physics
39	Seismology

APPENDIX A

LECTURES IN NUCLEAR CIVIL ENGINEERING

Delivered by James A. Cheney at

PUERTO RICO NUCLEAR CENTER
University of Puerto Rico
Mayaguez, P. R.

CONSTRUCTION TECHNOLOGY

Introduction

Construction is the ultimate object of design. In many cases, as in the peaceful uses of atomic explosions, the construction technique is the subject of research and development. Design must take into account these new construction methods and techniques.

The techniques proposed by the Plowshare project at Livermore Radiation Laboratory require projects of incredible magnitude in order to be economically feasible.

Building an Isthmian Canal by a single row of charges, building a harbor basin in one explosion, and others, require immense energy release and are only feasible because such an explosive, namely the nuclear explosive, is available.

The purpose of the Plowshare Summer Workshop is to see how this new technique fits into the construction industry as a new method in competition with existing methods of construction.

The Engineer and Construction

The engineer is that person who prepares the plans and specifications and supervises the construction of a project. In the case of the usual design project, the engineer is constantly concerned with reduction in costs if he is performing his service conscientiously. However, he is faced with the problem that cheaper, more efficient techniques of construction are sometimes not effected because the construction contractor does not recognize the economies of the new construction technique or unusual design. Contractors will bid high on things unusual or uncommon because they fear the unknown.

Usually when a project involves many uncertainties, the contract may be awarded on the basis of cost plus a fixed fee. Undoubtedly this would be the case in the Plowshare applications. But this leads to an unfair comparison in that the project by conventional methods would be by free competitive bidding. The former method has the disadvantage that the contractor is not concerned with keeping the costs down.

Costs may be considered divided into five items, materials, labor, equipment, overhead, supervision and profit. The last is not controlled by the engineer, but the others are influenced by the engineer and require that he is knowledgeable of construction methods. The Plowshare engineer is faced with the problem of comparing a new technique with current techniques. Some knowledge of current techniques is needed.

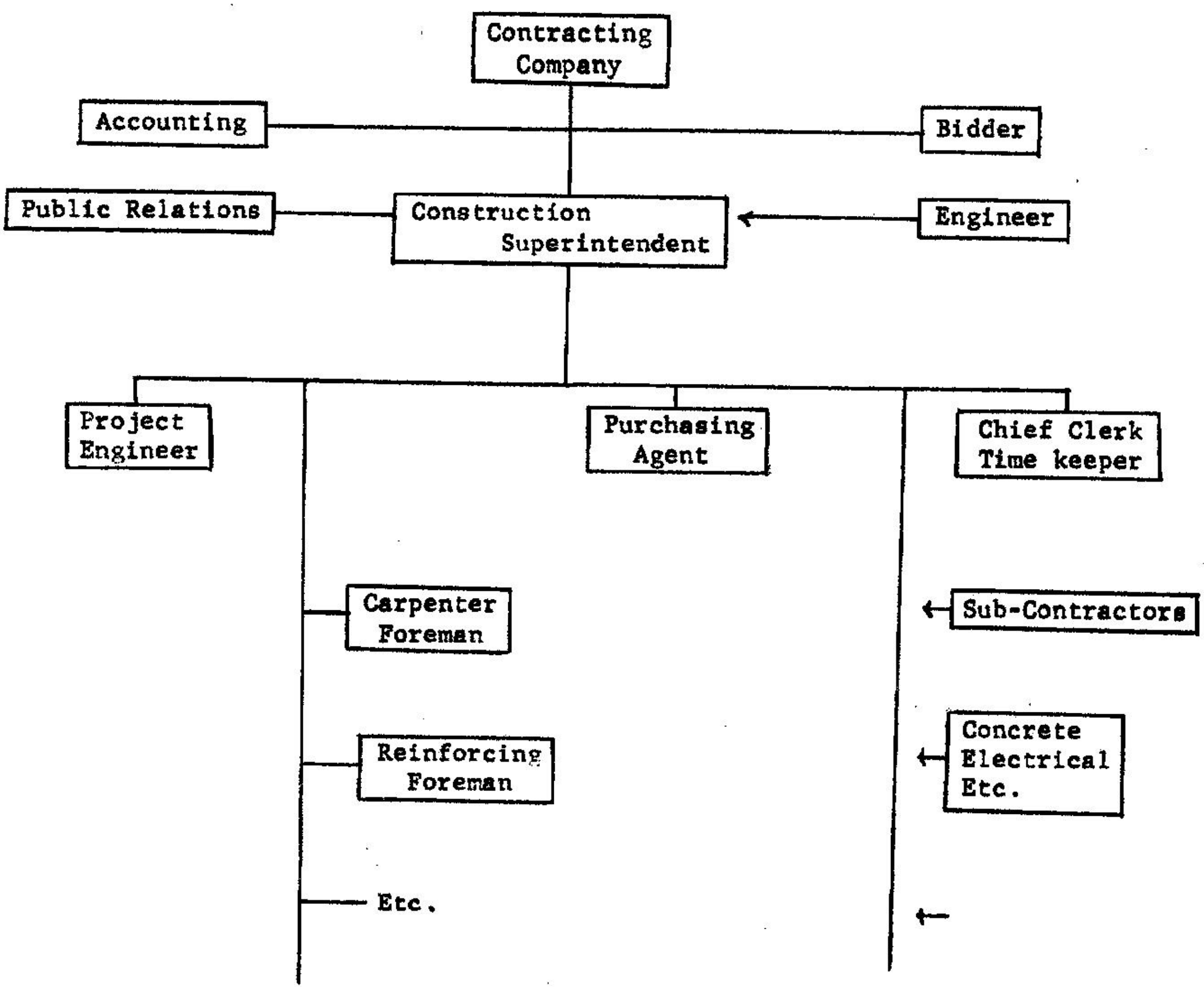
Areas of Knowledge

A basic knowledge of construction principles, soil mechanics, soil dynamics and nuclear explosion phenomenology is required for the simplest of cost and feasibility evaluations.

Construction Organization

The organization of a construction company in the United States of North America consists in most cases of a single contractor who employs a small group of trusted people who may include a bidder, a foreman and a time keeper and accountant. Larger organizations, which would be more likely to embark on a project involving nuclear power in construction, have administrative groups in their main office that carry out the same work.

A typical organization may be shown in an organization chart, as is often done in industry.



CONTRACTOR ORGANIZATION CHART

There are problems that merit discussion in each of the areas in block diagram above. The features which bear discussion involve:

Public Relations

Labor Law

Labor relations

Performance Bonds (Insurance)

Public Liability Insurance

Hidden Costs:

Lost Time

Cost of bidding

Miscellaneous insurance

Depreciation

Interest of capital investment

Accounting methods and cost control

Safety programs

The discussion as applied to Nuclear Civil Engineering will be centered on one area - that of bidding. This involves knowledge of all the other areas of management as they affect cost and also requires knowledge of the construction scheduling. A good bid will include a project schedule which should be used to determine the relative progress during construction.

In preparing a bid for a construction project, prebidding studies are required to determine the influence of:

Topography

Geology

Climate

Sources of Materials

Access to the project

Housing facilities

Storage facilities for materials and equipment

Labor supply

Local services

In the case of a Plowshare project many, if not all, living accomodations may have to be supplied by the contractor.

The use of substitute construction equipment having higher capacity, high efficiencies, higher speeds, more maneuverability, and lower operating costs should be considered. Bonuses to key personnel for beating dead lines, use of radios for quick communication, and periodic conferences with key personnel to discuss plans, procedures, and results, should aid morale and result in better coordination among various operations.

The Project Schedule

Until just a few years ago, there was no generally accepted formal procedure to aid in the management of projects. Each contractor had his own scheme, which often involved limited use of bar charts. The first formal procedure for determining a project schedule was PERT (Program Evaluation and Review Technique) (1956) and closely followed by CPM (Critical Path Method) (1959). The first was developed at Lockheed Missile and Space Co. in connection with the development of the Polaris weapon system and the latter was developed by the DuPont Company. Since that time the methods have been applied largely throughout the construction industry.

The principal feature of PERT is a statistical treatment of the uncertainty in activity performance time and includes an estimate of the probability of meeting specified scheduled dates at various stages in the project. The object of CPM is to determine how best to reduce the time required to perform routine construction work.

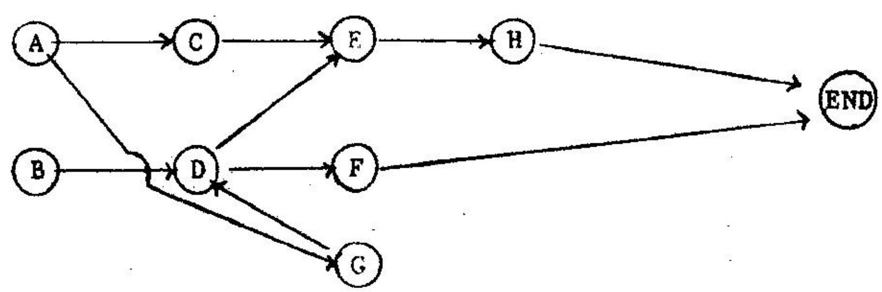
The nuclear civil engineering project is more likely to involve the CPM approach, although certain development aspects of the devices themselves and the determination of the scaling laws or computer codes might be appropriate ground for PERT procedures.

The Project Net Work

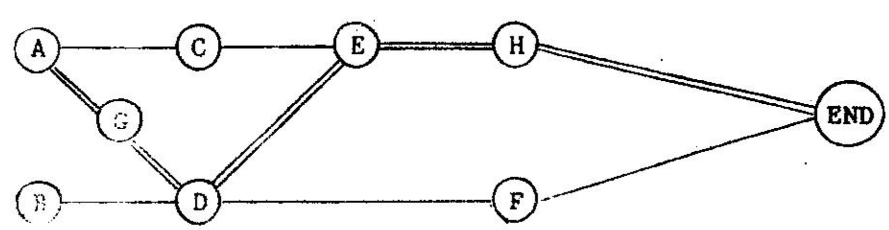
The project network is the heart of the CPM and PERT procedures. It is best explained by an example. We first start with statements which describe the sequence of operations making up a project. This may be given in the form of a chart.

Activity	Depends on	Activity
A		None
B		None
C		A
D		B G
E		C D
F		D
G		A
H		E

The node or circle notation places the activity network junctions. The first attempt at making a network is best done by connecting the operation, denoted by circles, with directed arrows indicating fine sequence. Thus for the above example.



Later this may be cleaned up to give a neater appearance and the arrow heads may be dropped since sequence may be associated with position.



Determination of the Critical Path

To determine the critical path the activities must be given times of completion. Usually the times given by a contractor would be those for the most economical production and, therefore, represent the minimum cost. We will call these costs "normal" costs. The following table will be made and described below:

Operation	Normal Time	E.S.	E.F.
A	5	0	5
B	10	0	10
G	10	5	15
C	20	5	25
D	15	15	30
E	20	30	50
F	10	30	40
H	7	50	57

E. S. and E. F. refer to the Earliest Start time and the Earliest Finish time for each operation. These times are determined by considering the normal time for the operation and the project network. A and B depend on nothing, therefore, start at zero. G, however, may not start until A is finished (see the network) therefore, the E. S. for G is the E. F. for A. If the start time for an operation depends upon more than one operation being finished, the later finish time of all preceding operations must be used. Thus, the E. S. for D is the E. F. for G rather than B. The procedure leads to the minimum number of days to complete the project, 57.

L. S. and L. F. refer to the Latest Start time and Latest Finish time that an operation may have and still complete the project in earliest possible time, i.e., 57 in the example. These numbers are obtained by starting at the last operation and determining how late it may be started and moving upward to determine this for each operation.

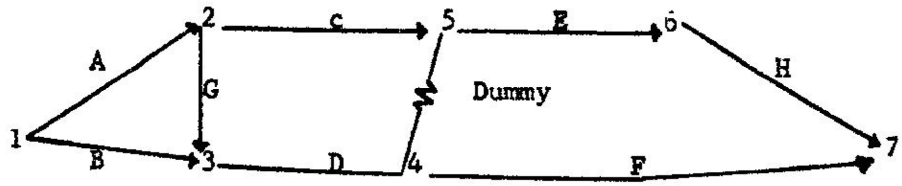
Operation	Normal Time	E.S.	E.F.	L.S.	L.F.	LAG	Cost
A	5	0	5	0	5	0	1,000
B	10	0	10	5	15	5	3,000
G	10	5	15	5	15	0	5,000
C	20	5	25	10	30	5	20,000
D	15	15	30	15	30	0	30,000
E	20	30	50	30	50	0	10,000
F	10	30	40	47	57	17	2,000
H	7	50	57	50	57	0	1,000
							$72,000

For example, F may finish at 57 along with H and, therefore, have a start at 47. E just finish before H so E has Latest Finish that is Latest Start for H. Where there is a choice between the start of two or more operations, the earlier of them must be used for the Latest Finish of the preceding operation.

The lag time is defined as either the difference between L.S. and E.S. or L.F. and E.F. Those with zero lag time are said to be critical and the network path connecting critical operations is a critical path. Thus, operations AGDEH form the critical path. The significance of the critical path is that in order to shorten the total time for a project some member of the critical path must be shortened.

Arrow Notation

The above analysis has used the circle notation in the project network. It should be pointed out that another notation is commonly used for application with the high speed digital computer. The operations are represented by arrows with each arrow represented by a number at its tail and a larger number at its head. Operations that have the same preceding operation carry the start number corresponding to the finish number of the preceding arrow. Thus, the example above may be depicted as:



An anomaly occurs where operation E follows C and D, but F follows just D. If E and F both had the same start numbers there would be no way of showing the proper sequence. The anomaly is solved by inventing an operation called Dummy which takes zero time, but separates the start of E from the end of D.

Shortening the Project

Since the normal time schedule was defined as giving minimum cost, shortening the project (or for that matter, lengthening the project) will increase the cost of production. Presumably there will be maximum cost associated with every operation being performed as fast as possible. The operation times for that case are called crash time and the associated cost the crash cost. A critical path analysis may be made for the crash program and is given below:

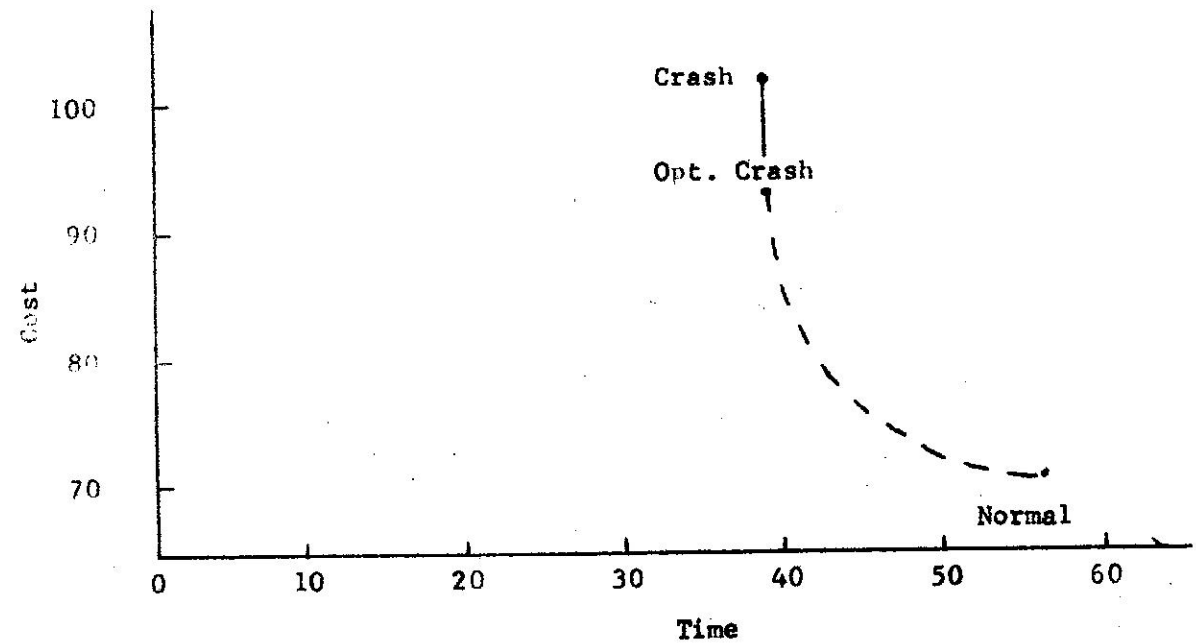
Critical Path for Crash Program

Operation	Normal Time	E.S.	E.F.	L.S.	L.F.	LAG	Cost
A	5	0	5	0	5	0	1,000
B	8	0	8	2	10	2	4,000
G	5	5	10	5	10	0	2,500
C	8	5	13	12	20	7	7,000
D	10	10	20	10	20	0	30,000
E	10	20	30	20	30	0	40,000
F	15	20	35	20	35	0	15,000
H	5	30	35	30	35	0	1,200
							$100,700

We may note immediately that those operations in the crash program which have lag time would be done foolishly since their time could be extended thus reducing cost without changing the duration of the project. The procedure we shall follow is to interpolate linearly between crash and normal time for the cost changes.

Project Shortening

Three points on a cost-time plot are available to us from the above analysis. Namely, the normal time, crash time and the optimum crash time. The optimum crash time cost is obtained by lengthening those operations which are not on a critical path to eliminate the lag times.



First we list the cost/day of shortening in order of increasing cost.

Operation	Cost/Day
G	100
H	100
B	500
C	1,000
D	1,000
E	1,000
F	2,000

To shorten the project in the least expensive way, we shorten the first operation on the list that is also on the critical path. This happens to be G or H. There are two restrictions on the amount of shortening. Either the operation is shortened to crash time or the lag time is used up in a parallel path whichever is smaller. This later limitation is determined by noting those operations whose lag time could be reduced by a reduction in operation time of G. Thus, operation lag times on C and B would be reduced by one day for every one day shortening of G. Since the maximum shortening of G (crash) is 6 days, but B has a lag equal to only 5, G may be shortened by 5 days and increased in cost by $500. This gives another point on the cost/time curve.

The next one to shorten is H. H has a crash limit of 2 days, but it affects only F which has a lag of 2, so the project is shortened by 2 days at an increase in cost of 200. The next cheapest reduction is either B and G, or D. (B is on the critical path now, but it can only be shortened if A or G is shortened too, but A cannot be shortened.)

Operation	Cost/Day
B G	500 100 $600
D	$1,000

The combination B and G has a smaller cost slope, but can only be shortened to crash time for G, which is 1 day at an increase in cost of $600. D may now be shortened to crash time leaving a lag of 4 in operation C, reducing the time by 10 days at a cost of $10,000. We thus arrive at the optimum crash program from the normal time point of the cost time curve.

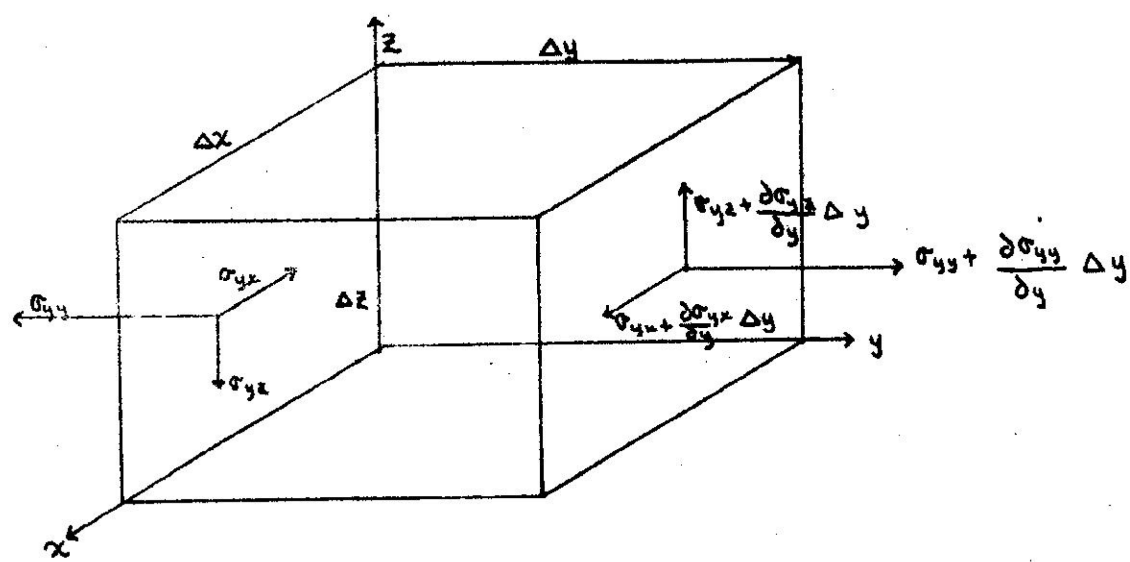
FUNDAMENTALS OF ELASTIC THEORY

Concept of the Continuum

The continuum is an assumption that macroscopic properties of materials are correct for microscopic elements of the object. This assumption permits the elimination of arguments concerning the interaction of repulsive and attractive forces between atoms which when viewed from the gross scale may be treated as average properties.

Equations covering an element

The equations that we write depend upon the coordinate system used. In Cartesian coordinates the element is rectangular.



The letter σ shall be used to denote stress. A stress is positive on a back face if it is directed opposite to the coordinate direction and positive on a front face if it is directed in the coordinate direction. The first subscript defines the face upon which the stress is acting and the second subscript the direction of the stress component. In order to not clutter the figure we have shown the stresses on two parallel planes. There are similar sets of stresses on the other planes. For each of the coordinate directions we may write the summation of forces. For example:

$$\left(\sigma_{yy} + \frac{\partial \sigma_{yy}}{\partial y}\Delta y\right)\Delta x \Delta z - \sigma_{yy}\Delta x \Delta z$$

$$+ \left(\sigma_{zy} + \frac{\partial \sigma_{zy}}{\partial z}\Delta z\right)\Delta x \Delta y - \sigma_{zy}\Delta x \Delta y$$

$$+ \left(\sigma_{xy} + \frac{\partial \sigma_{xy}}{\partial x}\Delta x\right)\Delta y \Delta z - \sigma_{xy}\Delta y \Delta z = \rho \Delta x \Delta y \Delta z \frac{\partial^2 v}{\partial t^2} - Y$$

where v is the displacement in the y direction. Y is a body force.

In addition to the sum of the forces we may also set the sum of the moments about any axis equal to zero. For example:

$$\left(\sigma_{yz} + \frac{\partial \sigma_{yz}}{\partial y}\Delta y\right)\Delta x \Delta z \Delta y - \left(\sigma_{zy} + \frac{\partial \sigma_{zy}}{\partial z}\Delta z\right)\Delta x \Delta y \Delta z$$

$$+ \frac{\partial \sigma_{yy}}{\partial y}\Delta y \Delta x \Delta z \frac{\Delta z}{2} + \frac{\partial \sigma_{zz}}{\partial z}\Delta z \Delta x \Delta y \frac{\Delta y}{2} = 0$$

Neglecting terms of higher order we obtain

$$\sigma_{yz} = \sigma_{zy}$$

Similarly

$$\sigma_{xy} = \sigma_{yx}$$

$$\sigma_{xz} = \sigma_{zx}$$

The above relations may be combined with the other equilibrium equations to give

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2} - X$$

$$\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + \frac{\partial \sigma_{xy}}{\partial x} = \rho \frac{\partial^2 v}{\partial t^2} - Y$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = \rho \frac{\partial^2 w}{\partial t^2} - Z$$

These are the equations of motion in terms of stress. If acceleration is zero they become the static equations of equilibrium.

Stress-Strain Relations

The constitutive relations (Equations of State) relate the stresses to appropriate strains. We shall use Hooke's law for these stress-strain relations:

$$E\epsilon_{xx} = \sigma_{xx} - \gamma(\sigma_{yy} + \sigma_{zz})$$

$$E\epsilon_{yy} = \sigma_{yy} - \gamma(\sigma_{xx} + \sigma_{zz})$$

$$E\epsilon_{zz} = \sigma_{zz} - \gamma(\sigma_{xx} + \sigma_{yy})$$

$$G\epsilon_{xy} = \sigma_{xy}$$

$$G\epsilon_{yz} = \sigma_{yz}$$

$$G\epsilon_{xz} = \sigma_{xz}$$

where $G = \dfrac{E}{2(1+\gamma)}$

Definition of Strain

To complete the set we must know the relations between strain and displacement:

$$\epsilon_{xx} = \frac{\partial u}{\partial x}\ ; \quad \epsilon_{yy} = \frac{\partial v}{\partial y}\ ; \quad \epsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\ ; \quad \epsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\ ; \quad \epsilon_{yz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}$$

These equations are sometimes called the kinematic relations.

Compatibility Relations

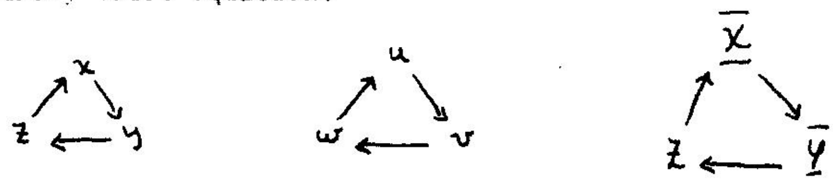
It may be noted that not all the strains can be independent since they are related to only three independent displacements. For example

$$\frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} = \frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2}$$

There are six such relations possible.

Cyclic-Substitution

It is often helpful to note that any equation in elasticity theory may be changed by a cyclic exchange of subscripts and displacements to yield another equally valid equation.



Re-arrangement of Equations

For the purpose of investigating the existence of elastic waves, we will rearrange the above three sets of equations (Equations of Motion, Equations of State, Kinematic Relations). The Equations of State (stress-strain law) may be solved for stresses:

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_{xx} + \nu\epsilon_{yy} + \nu\epsilon_{zz}\right]$$

$$\sigma_{yy} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_{yy} + \nu\epsilon_{xx} + \nu\epsilon_{zz}\right]$$

$$\sigma_{zz} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_{zz} + \nu\epsilon_{xx} + \nu\epsilon_{yy}\right]$$

$$\sigma_{xy} = G\epsilon_{xy}$$

$$\sigma_{yz} = G\epsilon_{yz}$$

$$\sigma_{xz} = G\epsilon_{xz}$$

or

$$\sigma_{xx} = \frac{\nu E}{(1+\nu)(1-2\nu)}\left(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}\right) + \frac{E}{1+\nu}\epsilon_{xx}$$

$$\sigma_{yy} = \frac{\nu E}{(1+\nu)(1-2\nu)}\left(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}\right) + \frac{E}{1+\nu}\epsilon_{yy}$$

$$\sigma_{zz} = \frac{\nu E}{\quad}\left(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}\right) + \frac{E}{1+\nu}\epsilon_{zz}$$

For convenience in writing some new symbols may be defined:

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad e = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$$

and

$$G = \frac{E}{2(1+\nu)}$$

Then

$$\sigma_{xx} = \lambda e + 2G\epsilon_{xx}$$
$$\sigma_{yy} = \lambda e + 2G\epsilon_{yy}$$
$$\sigma_{zz} = \lambda e + 2G\epsilon_{zz}$$
$$\sigma_{xy} = G\epsilon_{xy}$$
$$\sigma_{yz} = G\epsilon_{yz}$$
$$\sigma_{xz} = G\epsilon_{xz}$$

These equations for stress may be substituted into the equations of motion, resulting in

$$(\lambda + G)\frac{\partial e}{\partial x} + G\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \rho\frac{\partial^2 u}{\partial t^2} = 0$$

$$(\lambda + G)\frac{\partial e}{\partial y} + G\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \rho\frac{\partial^2 v}{\partial t^2} = 0$$

$$(\lambda + G)\frac{\partial e}{\partial z} + G\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \rho\frac{\partial^2 w}{\partial t^2} = 0$$

These are in the absence of body forces, X, Y or Z. These equations may be further simplified by use of the ∇^2 operator.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$(\lambda + G)\frac{\partial e}{\partial x} + G\nabla^2 u - \rho\frac{\partial^2 u}{\partial t^2} = 0$$

$$(\lambda + G)\frac{\partial e}{\partial y} + G\nabla^2 v - \rho\frac{\partial^2 v}{\partial t^2} = 0$$

$$(\lambda + G)\frac{\partial e}{\partial z} + G\nabla^2 w - \rho\frac{\partial^2 w}{\partial t^2} = 0$$

Waves of Dilatation

Differentiate the three equations respectively with respect to x, y and z and then add them together. We obtain

$$(\lambda + 2G)\nabla^2 e - \rho \frac{\partial^2 e}{\partial t^2} = 0$$

This defines the propagation of a disturbance of volume. The above relationship may also be derived on the basis of considering the type of motion which is free of rotation. Hence, the waves resulting are considered irrotational, dilatational or simply P (waves).

Shear Waves

If we place the restriction of no change in volume on the differential equations, ϵ = constant and

$$G\nabla^2 u - \rho \frac{\partial^2 u}{\partial t^2} = 0$$

$$G\nabla^2 v - \rho \frac{\partial^2 v}{\partial t^2} = 0$$

$$G\nabla^2 w - \rho \frac{\partial^2 w}{\partial t^2} = 0$$

The above expressions do permit shear distortion; hence, these disturbances are called equivoluminal, shear, or S waves.

Velocity of Propagation

The wave equation has the form

$$\frac{\partial^2 f}{\partial t^2} = C^2 \nabla^2 f$$

wherein C is the velocity of propagation. Hence, the dilatational wave has velocity

$$C_d = \sqrt{\frac{\lambda + 2G}{\rho}}$$

and the shear wave has velocity

$$C_s = \sqrt{\frac{G}{\rho}}$$

The dilatational waves travel faster than the shear waves, hence the wave fronts of these two waves move farther and farther apart as they move from a source. Given a specified distance from the source R

$$R = t_d C_d = t_s C_s$$

$$\Delta t = t_s - t_d$$

then

$$\Delta t = \frac{R}{C_s} - \frac{R}{C_d} = \frac{C_d - C_s}{C_s C_d} R$$

If the time difference between the arrival of the dilatational wave and the shear wave is measured

$$R = \Delta t \frac{C_d C_s}{C_d - C_s}$$

This relationship is used to determine the distance of an earthquake from an observer. Several stations are required to pin point the actual location. The determination depends upon the accurate knowledge of C_d and C_s for the earth.

The dilatational wave arrives first, hence is called primary (P), and the shear wave arrives second, hence secondary (S). For example, in steel C_d = 18,000 fps, C_s = 10,500 fps, whereas in granite C_d = 16,000 fps and C_s = 10,000 fps.

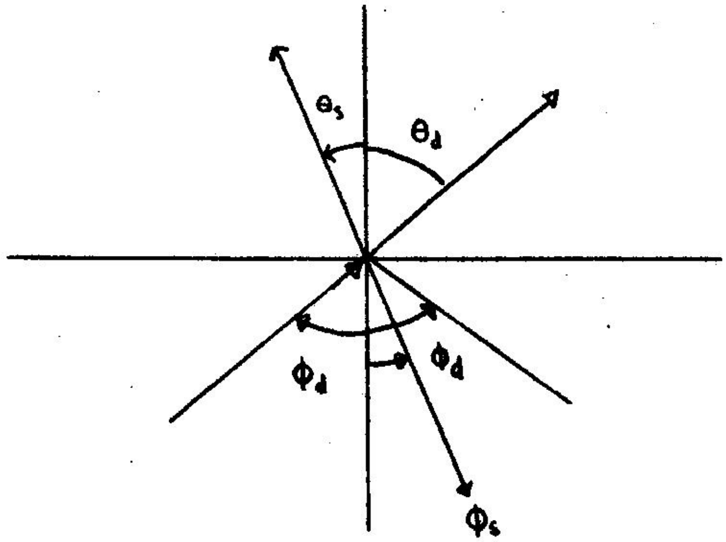
Surface Waves

Waves are reflected from a surface of discontinuity in the medium. In general, the incident wave produces four waves: a reflected dilatational wave and shear wave, and a refracted pair. Of course, amplitudes and angles with respect to the surface of the subsequent waves must be such that the stress components across the surface be continuous.

From the requirement of continuity of displacement

$$\frac{1}{C_d}\sin\phi_d = \frac{1}{C_s}\sin\phi_s = \frac{1}{C'_d}\sin\theta_d = \frac{1}{C'_s}\sin\theta_s$$

wherein ϕ_d and ϕ_s are angles of reflection and θ_d and θ_s are angles of refraction to the angle of incidence ϕ_d



When waves from the interior strike a free surface a special type of wave occurs which has been called the Rayleigh wave. If the x-z plane is assumed to coincide with the free surface, and propagation is assumed to be in the x direction, W=0 is implied. Since neither dilatational or shear waves satisfy these conditions, a new type of wave must be developed. Actually the surface wave developed by Lord Rayleigh is a combination of the two waves previously described. The derivation made by Lord Rayleigh will be outlined briefly and the results discussed.

The following expressions for dilatational waves were assumed:

$$u_d = s e^{-ry} \sin(pt - sx), \quad v_d = -r e^{-ry} \cos(pt - sx)$$

$$w_d = 0$$

and the following for shear waves:

$$u_s = b e^{-by} \sin(pt - sx), \quad v_s = -s e^{-b} \cos(pt - sx)$$

$$w_s = 0$$

It can be shown that the expressions satisfy the conditions for the two respective waves.

A more general type of plane wave is obtained by linear combination of the above two waves, i.e.

$$u = s e^{-ry} \sin(pt - sx) + A b e^{-by} \sin(pt - sx)$$
$$v = -r e^{-ry} \cos(pt - sx) - A b e^{-by} \cos(pt - sx)$$
$$w = 0$$

The constants A, b, p, r, s have to be evaluated to satisfy the boundary conditions. These are to express mathematically that on the surface y = o, the normal stress ρ_{xy} and shearing stress ρ_{yz} are zero.

Using the expressions developed for stress we obtain the equations

$$\lambda e + 2G \epsilon_{yy} = 0$$
$$\epsilon_{xy} = 0$$
$$\epsilon_{yt} = 0$$

or in terms of displacement, with w = o

$$\lambda \left(\frac{\partial y}{\partial x} + \frac{\partial v}{\partial y}\right) + 2G \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

while $\epsilon_{yz} = 0$ is identically satisfied.

If the two sets of solutions are substituted into the dilatational wave equation and the shear wave equations, the following relationships must hold:

$$r^2 = s^2 - \frac{\rho p^2}{\lambda + 2G}$$

$$b^2 = s^2 - \frac{\rho p^2}{G}$$

The velocity of propagation in the x direction is

$$C_r = \frac{p}{s}$$

Substituting our proposed combined solution into the boundary conditions, eliminating the constant A, and introducing previously determined values of C_d and C_s, we obtain

$$\left(\frac{C_r^2}{C_s^2} - 2\right)^4 = 16\left(1 - \frac{C_r^2}{C_d^2}\right)\left(1 - \frac{C_r^2}{C_s^2}\right)$$

since

$$\frac{C_s^2}{C_d^2} = \frac{1-2\nu}{2(1-\nu)}$$

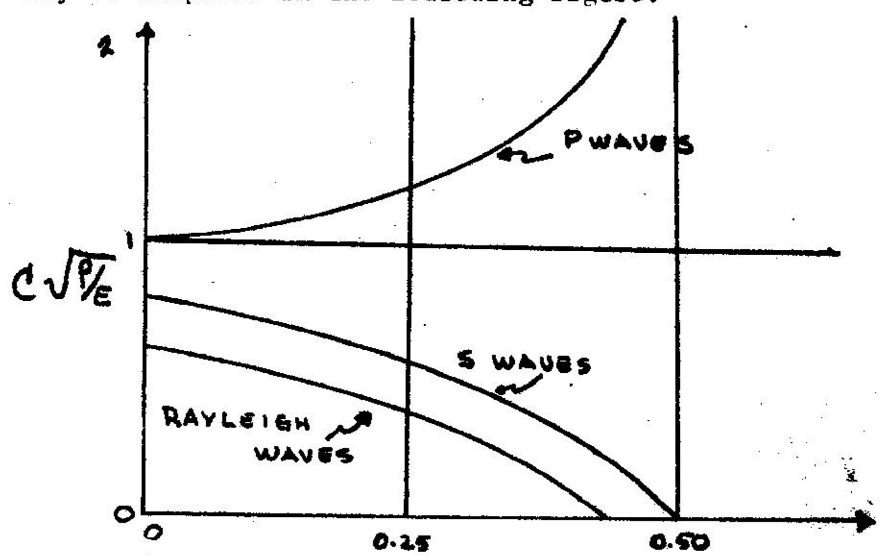
and taking

$$\frac{C_r}{C_s} = \alpha$$

we obtain

$$\alpha^6 - 8\alpha^4 + 8\left(3 - \frac{1-2\nu}{1-\nu}\right)\alpha^2 - 16\left(1 - \frac{1-2\nu}{2(1-\nu)}\right) = 0$$

The roots of this equation define the velocities of propagation. The results may be compared in the following figure.



Thus, we see that Rayleigh waves always are slower in velocity of propagation than the other two types.

FAILURE THEORIES

The Stress Tensor

We have already dealt with the stresses acting on an elemental cube. The entity of all stresses associated with one point regardless of orientation of the element is called the stress field. It may be expressed as a matrix or as a tensor.

The matrix form:

$$S = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yz} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

The tensor form is simply σ^{ji}

An entity is a tensor if it transforms according to a certain transformation law. Namely, if we rotate the coordinate system by an angle θ

$$\sigma_{x'x'} = \sigma_{xx} \cos^2\theta + \sigma_{yy} \sin^2\theta + 2\sigma_{xy} \cos\theta \sin\theta$$

$$\sigma_{x'y'} = (\sigma_{yy} - \sigma_{xx}) \cos\theta \sin\theta + \sigma_{xy} (\cos^2\theta - \sin^2\theta)$$

$$\sigma_{y'y'} = \sigma_{xx} \sin^2\theta + \sigma_{yy} \cos^2\theta - 2\sigma_{xy} \cos\theta \sin\theta$$

where in X^1, y^1 are the new coordinate directions.

Note that a vector transforms simply a

$$T_{x'} = T_x \cos\theta + T_y \sin\theta$$

$$T_{y'} = -T_x \sin\theta + T_y \cos\theta$$

or in matrix notation

$$\begin{bmatrix} T_{x'} \\ T_{y'} \end{bmatrix} = \begin{Bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{Bmatrix} \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

For the stress vectors on the principal planes x and y, $F_{xx} = \sigma_{xx} A_x$, $F_{yx} = \sigma_{yx} A_y$, $F_{yy} = \sigma_{yy} A_y$, $F_{xy} = \sigma_{xy} A_x$

$$\begin{bmatrix} F_{x'x'} & F_{x'y'} \\ F_{y'x'} & F_{y'y'} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} F_{xx} & F_{xy} \\ F_{yx} & F_{yy} \end{bmatrix}$$

The reciprocal of the areas associated with stress vectors F also transform vectorially.

$$\begin{bmatrix} \frac{1}{A_{x'x'}} & \frac{1}{A_{y'x'}} \\ \frac{1}{A_{x'y'}} & \frac{1}{A_{y'y'}} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \frac{1}{A_x} & 0 \\ 0 & \frac{1}{A_y} \end{bmatrix}$$

$$[\sigma] = [F] \times \left[\frac{1}{A}\right]$$

which leads to

$$[\sigma'] = [a][F] \times [a]\left[\frac{1}{A}\right]$$

or

$$[\sigma'] = [a][F]\left[\frac{1}{A}\right][a]$$

wherein $[a] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ the vector transform and a^T is the transpose of a.

This may then be written simply.

$$[\sigma'] = [a][\sigma][a^T]$$

Any quantity that transforms with a rotation of the coordinate system as above is called a tensor. Essentially a tensor is an entity that is the product of two vectors.

Strain is a tensor quantity. The displacements if kept small are vector quantities (transforming a vector for rotations of the coordinate system). The lengths (sides of the element) are also vector quantities and the inverses of the lengths are also vector quantities leading to

$$[\epsilon'] = [a][\epsilon][a^T]$$

Stress Invariants and Yield Criteria

Certain quantities may be found which remain constant when a transformation is made. One of these is called the trace

$$\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 3\bar{\sigma}$$

The stress matrix may be written as

$$S = \begin{bmatrix} \bar{\sigma} & 0 & 0 \\ 0 & \bar{\sigma} & 0 \\ 0 & 0 & \bar{\sigma} \end{bmatrix} + \begin{bmatrix} \sigma'_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma'_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma'_{zz} \end{bmatrix}$$

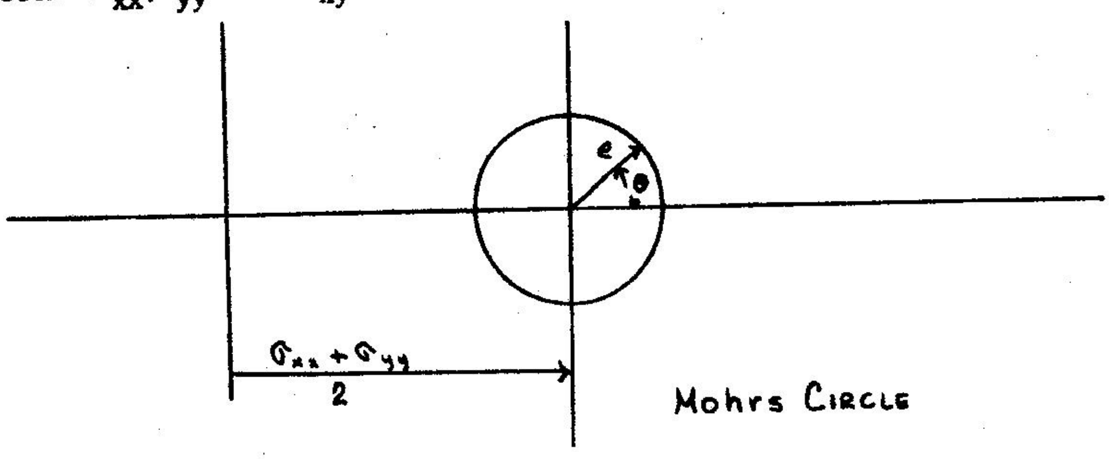
wherein $\sigma'_{xx} = \sigma_{xx} - \bar{\sigma}$ etc.

$\bar{\sigma}$ is a hydrostatic stress system.

Another invariant is

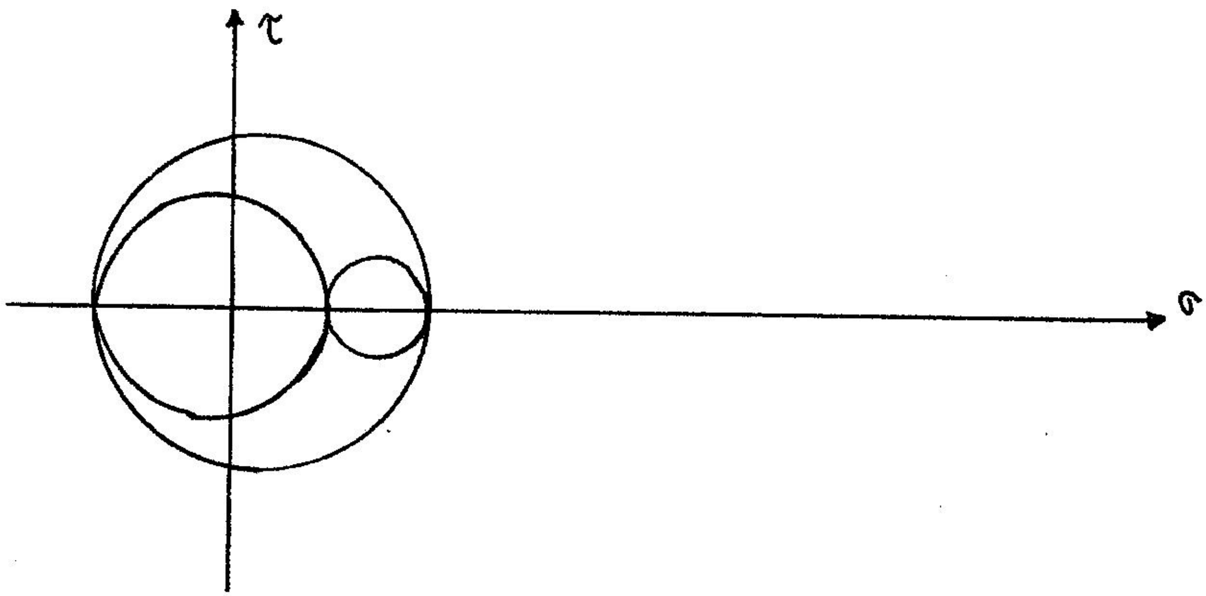
$$e^2 = \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2$$

This can be interpreted as the equation of a circle (Mohrs circle) in coordinate system σ_{xx}, yy and σ_{xy}.



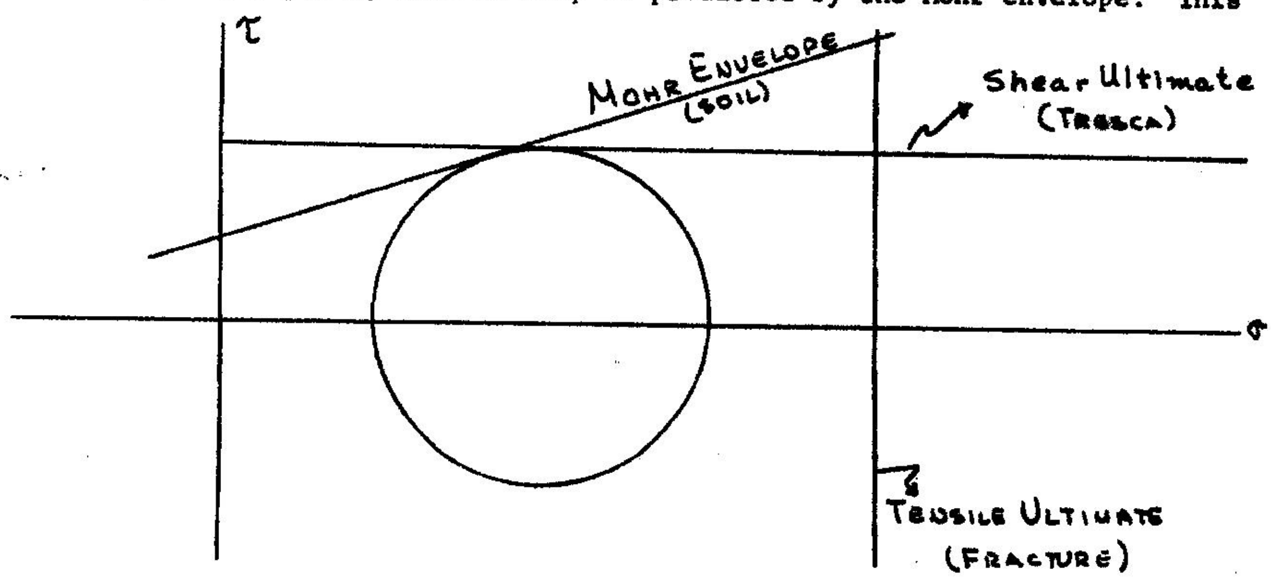
Mohrs Circle

wherein the angle θ represents the orientation with respect to the principle planes, i.e., those planes having zero shear stress. In general, 3 Mohrs circles can be drawn for rotations about the three coordinate directions:



Mohr Circle in Three Dimensional Stress

Failure may occur because material parts of slips. Thus, two limits may be drawn, one limiting the maximum shear, the other limiting the maximum tension. The maximum shear criterion for yielding is due to Tresca and is called the Tresca Yield criterion. The tensile limit is really a rupture criterion. Another type of rupture, that is slip is predicted by the Mohr envelope. This



forms the strength criterion for most soils. In general, the strength criterion for a material must be related to the stress invariants if the material is isotropic. A stress invariant is a relation involving the stress components that remains constant during a rotation of the coordinate system. In general, there are three independent stress invariants for isotropic materials.

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 = 3\bar{\sigma}$$

already mentioned.

$$J_2 = \frac{1}{2}\left[(\sigma_x - \bar{\sigma})^2 + (\sigma_y - \bar{\sigma})^2 + (\sigma_z - \bar{\sigma})^2\right] + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2$$

$$= \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right]$$

which may be related to the radius of the Mohr circle being constant

$$J_3 = \frac{1}{3}(\sigma_1 - \bar{\sigma})(\sigma_2 - \bar{\sigma})(\sigma_3 - \bar{\sigma})$$

wherein $\sigma_{1,2,3}$ are the principal stresses. The Mohr envelope criteria for yield implies that the mean normal stress is important. This may also be true in materials at extremely high pressure. Thus, a general yield theory involves all three stress invariants.

The most commonly used yield criteria are: <u>Tresca</u> (Maximum Shear Stress)

$$\sigma_1 - \sigma_3 = 2k$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$

<u>Von Mises</u>

$$J_2 = k^2$$

<u>Drucker and Prager</u>

$$\alpha J_1 + J_2^{1/2} = k$$

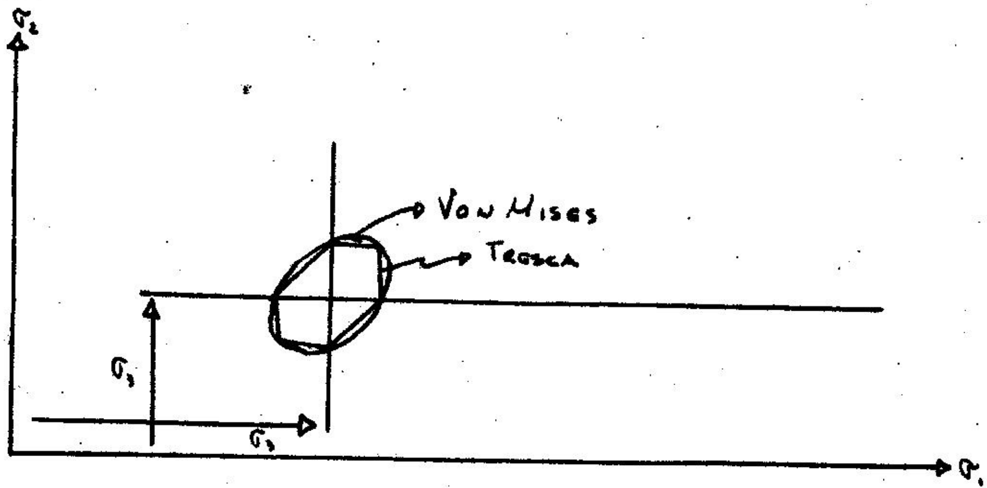
The latter being a generalization of the Mohr envelope or Coulomb's law for soils.

Osgood

$$J_2^3 - 2.25 J_3^2 = k^6$$

The latter was obtained empirically from tests on aluminum tubes.

The von Mises criterion was originally proposed as a mathematically smooth approximation of the Tresca criterion. In three dimensions using the principal stresses σ_1, σ_2, σ_3, we can plot the von Mises yield condition as a circular cylinder of radius $\sqrt{2}$ k whose axis is equally inclined to the three principal axes.



The Tresca yield criteria matches the von Mises criteria at six points, but is formed of straight interaction lines rather than curved.

Phenomenology of Stress-Strain-Time

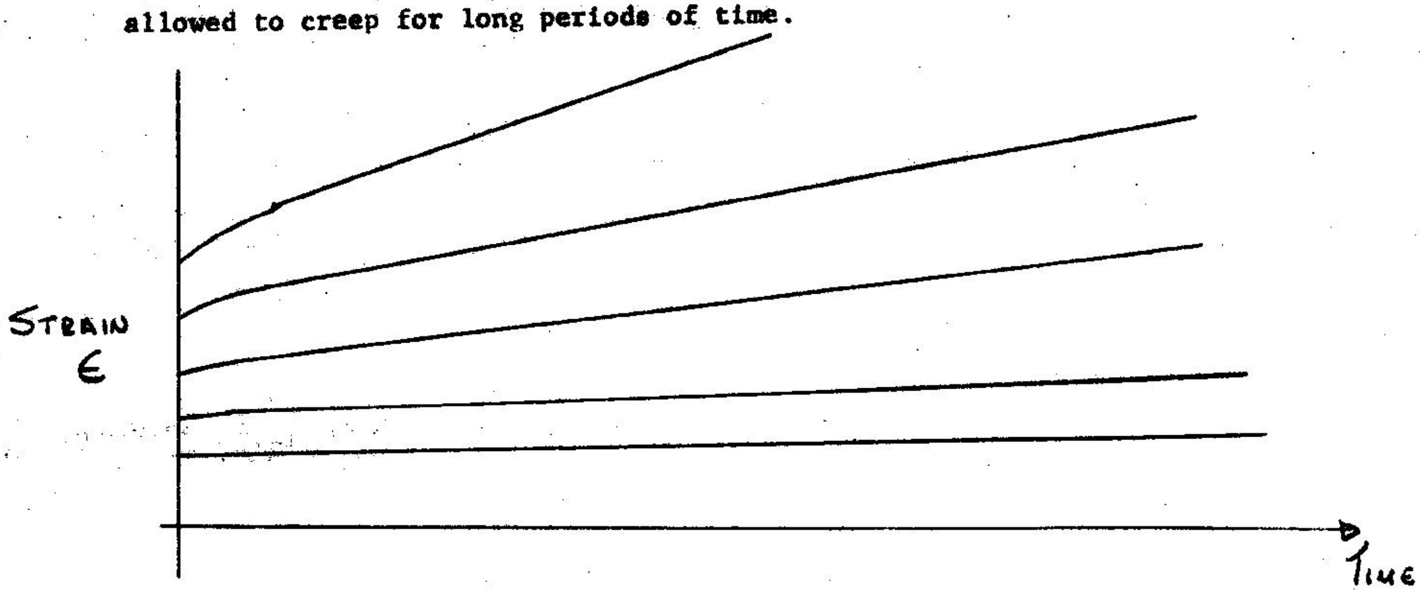
The foregoing criteria for yielding attempt to predict at what value of stress the stress-strain curve for the material subjected to biaxial and triaxial loading will become non-linear.

For many years work has been carried out to explain why after the yield point is reached and plastic action is occurring the stress continues to rise.

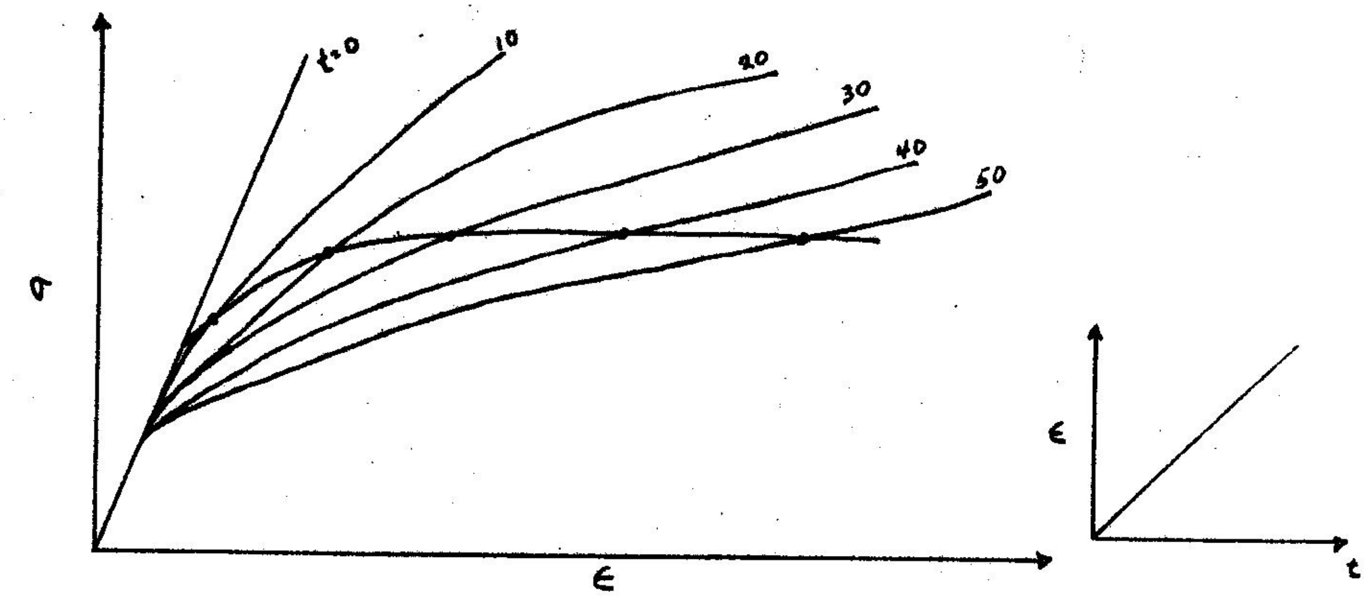
This behavior has been called work hardening. The reverse question, "why is it non-linear at all?" was not asked. It is the answer to this question that gives the most light to the subject.

If one asks what should the breaking strength of metals be, based upon the forces of attraction between particles, the answer from the physicist is that the breaking strength is in the order of Young's modulus, i.e., 10^7 psi. To answer why this strength is not achieved, a new theory was proposed called Dislocation Theory. The basis of dislocation theory is that, if an element in an array of elements is missing then as the array is distorted by shear, a certain strain is reached (elastically) where a neighboring element is likely to jump laterally to fill the void. Essentially the void or dislocation moves across the field until it reaches a boundary leaving behind a permanent set in the strain. A metal is visualized as being strewn with such dislocations that permit plastic flow. This plastic flow takes time to occur, however, and also probability enters into the occurrence of each dislocation jump.

From an engineering macroscopic view there are two types of behavior occurring with each addition of load: an elastic stretching which takes place instantaneously and a plastic or creep stretching which takes time. Creep tests can be run where the load is applied suddenly and the material then allowed to creep for long periods of time.



A cross plot of these curves for a given time since loading yields iso-chronous stress strain curves



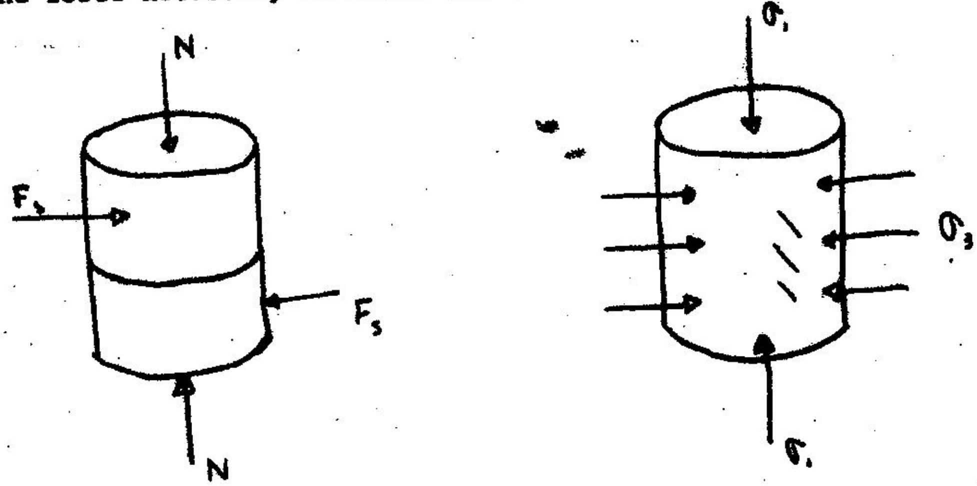
The usual stress strain diagram may be generated from these curves. The test machine generally operates at a constant strain rate. Once the stress level is above a plastic threshold, a certain amount of creep occurs depending upon time at the stress level for each loading step. We can approximate the behavior by connecting points on the isochronous stress strain diagrams corresponding to creep time and total strain.

The important point here is that the stress-strain diagram depends upon the rate of loading. The more rapid the loading the more elastic it behaves and may under extremely rapid loading behave as a brittle material and shatter.

SOIL MECHANICS

Strength

In the discussion of failure theories it was mentioned that the Mohr envelope or Coulomb law is used in Soil Mechanics. There are two types of soil tests used to determine the basic strength parameters in the Coulomb law: the direct shear and triaxial compression test. The direct shear test takes a block of the soil, places a normal load on the block and then determines the force necessary to shear the block in two.



The triaxial test takes a cylinder of the soil, usually sealed by a thin rubber membrane, places it in a hydrostatic pressure and then applies an axial load to failure. Mohr's circle can be drawn for either loading situation and enough variation made in the normal force or the hydrostatic pressure to define the Mohr envelope.

The Mohr envelope may be curved. However, for purposes of simplicity it is assumed to be linear in the range of interest, namely in the form

$$\tau_{CRITICAL} = c + \sigma_n \tan \phi_e$$

wherein τ critical surface is the shear strength on the failure

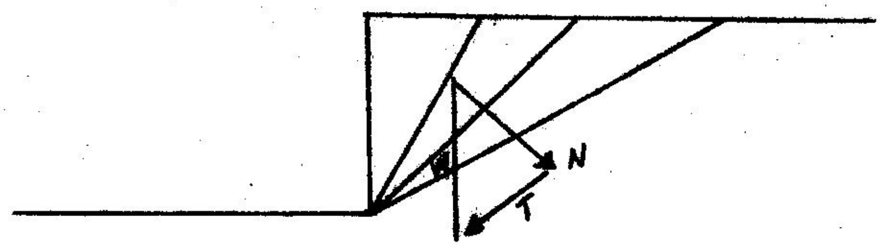
c is the cohesion

σ_n is the normal stress on the failure surface

$\tan \phi_e$ is the coefficient of internal friction
(ϕ_e the effective friction angle)

Rankine and Coulomb

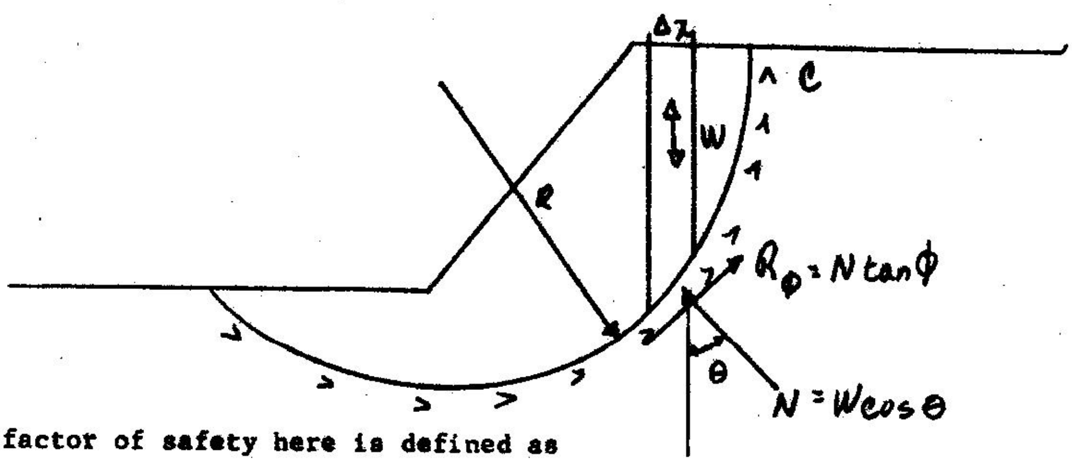
The earliest work done on slope stability and the stability of retaining walls was by Rankine and Coulomb. Essentially they used the above equation and assumed a straight sliding surface.



The surface that gave the lowest factor of safety is considered critical. The factor of safety is defined as

$$S.F. = \left(\frac{N \tan \phi e}{T}\right) \text{minimum}$$

Fellinius (in Sweden) noted that typical slope failures did not follow a straight failure surface but, rather, curved. He, therefore, suggested and used a circular arc of failure.



The factor of safety here is defined as

$$S.F. = \frac{\Sigma \text{ Resisting Moments}}{\Sigma \text{ Driving Moments}}$$

The slope is divided into vertical slices (hence the common name: Method of Slices) and the forces on the trial sliding surface are determined ignoring the possibility of interaction between the slices themselves. The W is determined as the volume of the slice times the unit moist weight. The local angle θ must be measured for each slice and the resisting moments summed

$$M_R = \sum_N \left[\frac{C\Delta x}{\cos\theta} + \Delta W \cos\theta \tan\phi\right] R$$

The driving moment may be determined by summation

$$M_d = \sum \Delta W R \sin\theta$$

Two other types of safety factor can be calculated:

Safety factor with respect to height:

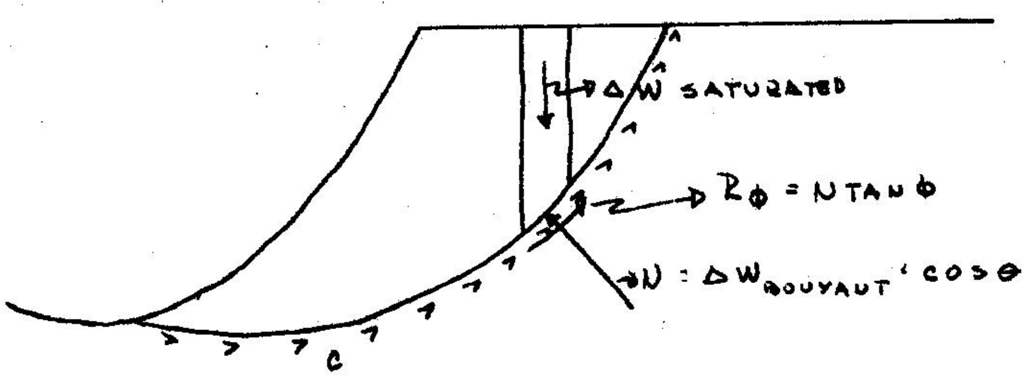
$$S.F. = \frac{\sum c \frac{\Delta x}{\cos\theta}}{\sum\left[\Delta W \sin\theta - \Delta W \cos\theta \tan\phi\right]}$$

Safety factor with respect to friction

$$S.F. = \frac{\sum \Delta W \cos\theta \tan\phi}{\sum \Delta W \sin\theta - c \frac{\Delta x}{\cos\theta}}$$

The Fellinius method in principle can be applied for any loading condition including seepage and ground acceleration.

Example: Consider the case of a saturated soil immediately after the formation of a slope by an explosive. The normal effective stress across a trial sliding surface will not have had time to change while the full saturated weight of soil acts in the driving moment.



FLOW THROUGH POROUS MEDIA

Darcy's Law

$$q = k\,i\,A$$

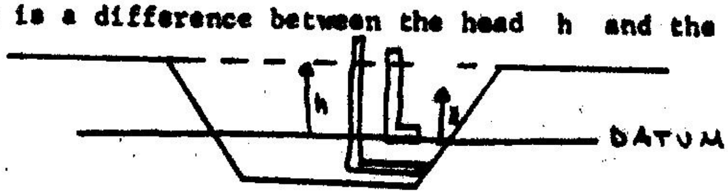
Wherein

- q = flow rate
- k = coefficient of permeability
- i = hydraulic gradient
- A = cross sectional area

The hydraulic gradient for incompressible fluids is given by

$$i = \frac{\partial h}{\partial s}$$

Note that there is a difference between the head h and the pressure head $\frac{p}{\gamma_o}$



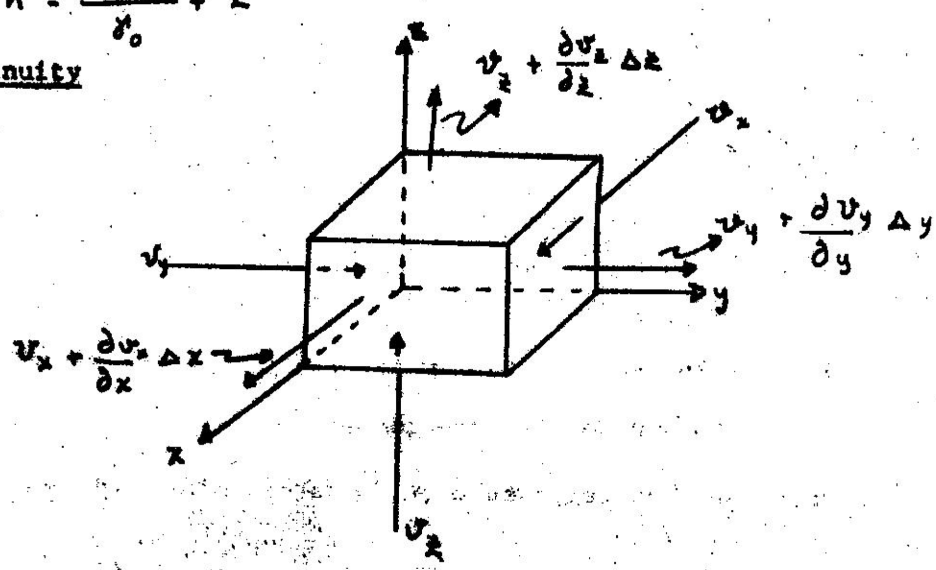
The pressure is given by

$$p = h\gamma_o - z\gamma_o$$

Hence in terms of p

$$h = \frac{p}{\gamma_o} + z$$

Continuity



Stuff in = Stuff out

Assume incompressible flow:

$$v_x \Delta y \Delta z + v_y \Delta x \Delta z + v_z \Delta x \Delta y =$$

$$\left(v_x + \frac{\partial v_x}{\partial x}\Delta x\right)\Delta y \Delta z + \left(v_y + \frac{\partial v_y}{\partial y}\Delta y\right)\Delta x \Delta x + \left(v_z + \frac{\partial v_z}{\partial z}\Delta z\right)\Delta x \Delta y$$

which reduces to

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Substituting $v_x = (Q/A)_x$ into the continuity

equation

$$k\frac{\partial i_x}{\partial x} + k\frac{\partial i_y}{\partial y} + k\frac{\partial i_z}{\partial z} = 0$$

or

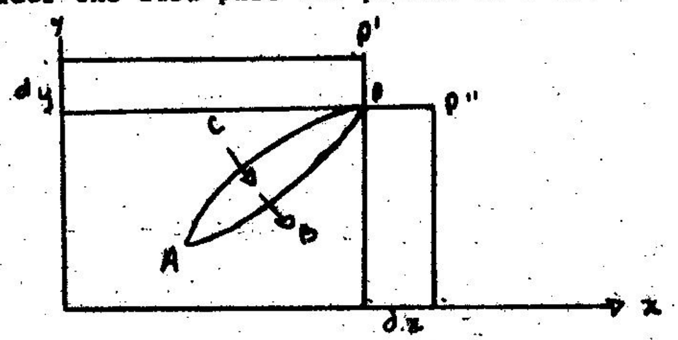
$$k\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) = 0$$

or

$$\nabla^2 h = 0 \qquad \textit{Laplace's Equation}$$

If we define our system in two dimensions the following argument can be made:

Consider the flow past two points in a two dimensional flow. The flow across



$\overline{ABP}$ is the same as the flow across $\overline{ACP}$ by continuity. Let the function denote the flow rate from left to right across any path connecting A and P. If we take another point p' the added flow rate would be Ψ' at P' minus Ψ at P or $\psi' - \psi$. Let PP' be dy, then

$$v_x \, dy = \frac{\partial \psi}{\partial y} dy$$

Similarly, for an element PP" = dx

$$-v_y \, dx = \frac{\partial \psi}{\partial x} dx$$

Hence

$$v_x = \frac{\partial \psi}{\partial y} \; ; \quad v_y = -\frac{\partial \psi}{\partial x}$$

But continuity requires that

$$\frac{\partial v_x}{\partial x} = -\frac{\partial v_y}{\partial y}$$

We see that ψ satisfies continuity identically. Thus the velocity components may be given by

$$v_x = k \frac{\partial h}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v_y = k \frac{\partial h}{\partial y} = -\frac{\partial \psi}{\partial x}$$

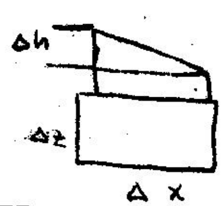
From this relation and from previous logic the stream lines (lines of constant ψ) are at right angles (orthogonal) to the equipotential lines (h = const.)

SKETCHING EQUIPOTENTIAL LINES

Consider the flow to be in the X direction in a system having a total head drop of H. The equipotential lines can be drawn so that each represents an equal drop Δh where: $\Delta h = \frac{H}{h}$

The hydraulic gradient is then

$$i = \frac{\Delta h}{\Delta x}$$



The velocity across the element is

$$v_x = k \frac{\Delta h}{\Delta x}$$

The flow is

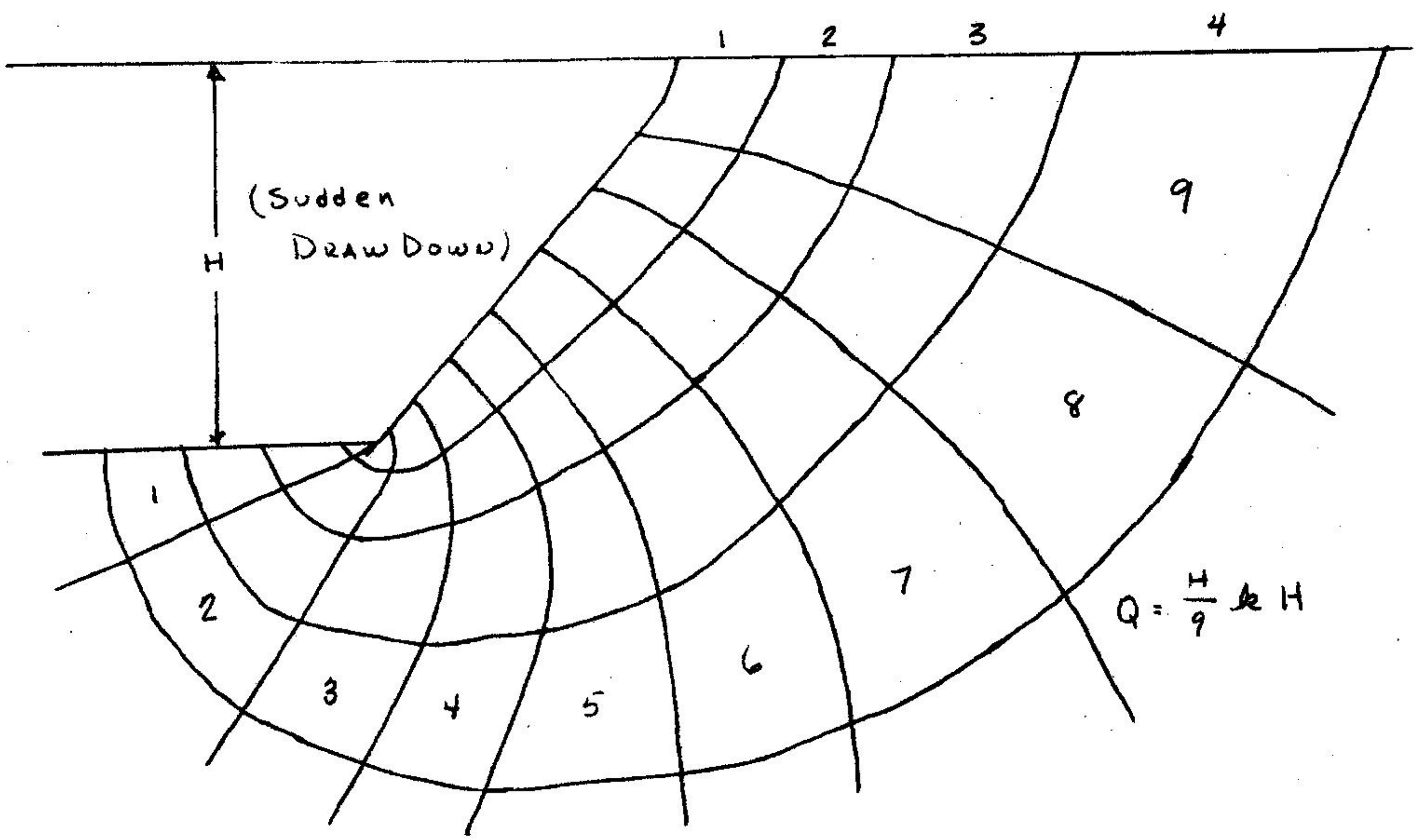
$$\Delta Q = v_x \, \Delta z = k \, \Delta h \, \frac{\Delta z}{\Delta x}$$

Make $\Delta z / \Delta x = 1.0$,

Then ΔQ in one flow path $= k \, \Delta h = k \, H/n$

Count the number of flow tubes, m

Total flow $\quad Q = m \, \Delta Q = \frac{m}{n} k H$



Relation Between Particle Velocity and Macroscopic Velocity

If we wish to know where a particular particle of water is after a length of time t, we must know the true velocity at all times. But this varies from point to point depending upon the constrictions through which the water flows.

One approach is to take an effective tube size that is equal to gross area (A) times the porosity (n) of the permeable material. Porosity being

$$n = \frac{V_v}{V_v + V_s}$$

where in V_v = volume of voids

V_s = volume of solids

In this way the average particle velocity is given by

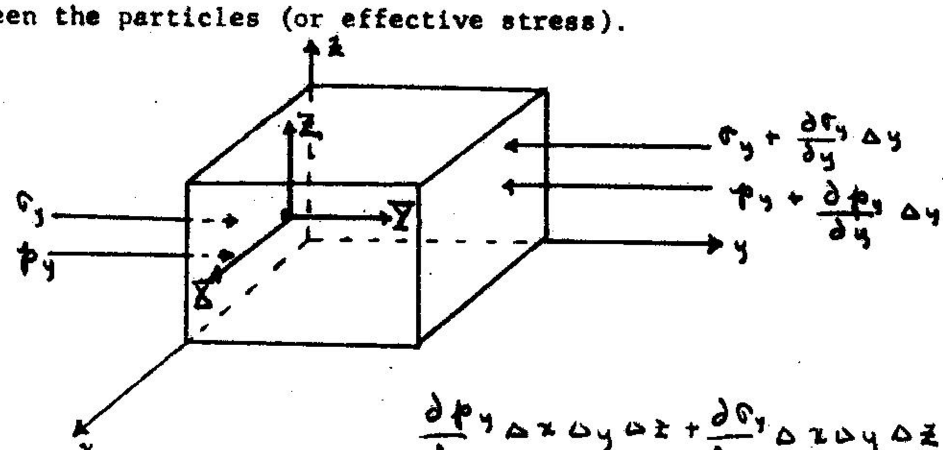
$$\bar{v} = \frac{u}{n}$$

<u>Coefficient of permeability</u>

The coefficient k may be written as coefficient $\bar{k}$ divided by the viscosity μ wherein $\bar{k}$ is a property of the porous material, having units of length squared and μ is a property of the fluid, having units of lb-sec/ft^2.

<u>Forces Due to Seepage</u>

There are two systems of forces: forces in the water phase, and forces between the particles (or effective stress).



$$\frac{\partial p_y}{\partial y}\Delta x \Delta y \Delta z + \frac{\partial \sigma_y}{\partial y}\Delta x \Delta y \Delta z = \bar{Y}$$

The body force $\bar{Y} = 0$, normally. In fact

$$\bar{X} = \bar{Y} = 0 \qquad \bar{Z} = \gamma_{submerged}\,\Delta z\,\Delta y\,\Delta x$$

Hence:

$$\frac{\partial p}{\partial y} = -\frac{\partial \sigma}{\partial y}$$

or

$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial y}(h - z)\gamma_o = \gamma_o \frac{\partial h}{\partial y} = \gamma_o i_y$$

therefore,

$$\frac{\partial \sigma}{\partial y} = \gamma_o i_y$$

Similarly

$$\frac{\partial \sigma}{\partial x} = \gamma_o i_x$$

However, for the vertical direction

$$\frac{\partial \sigma}{\partial z} = \gamma_o i_z - \gamma_{submerged}$$

Note that for a vertical flow we may reach a Quick Condition wherein $\sigma_z = 0$ everywhere. Then $\gamma_o i_z = \gamma_{sub}$ and $i_z = \gamma_{sub}/\gamma_o$

or $\bar{v}_{CRIT.} = \frac{k}{h} \gamma_{sub}/\gamma_o$

<u>Anisotropic Permeability</u>

$$v_x = k_x \frac{\partial h}{\partial x}, \quad v_y = k_y \frac{\partial h}{\partial y}, \quad v_z = k_z \frac{\partial h}{\partial z}$$

then

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

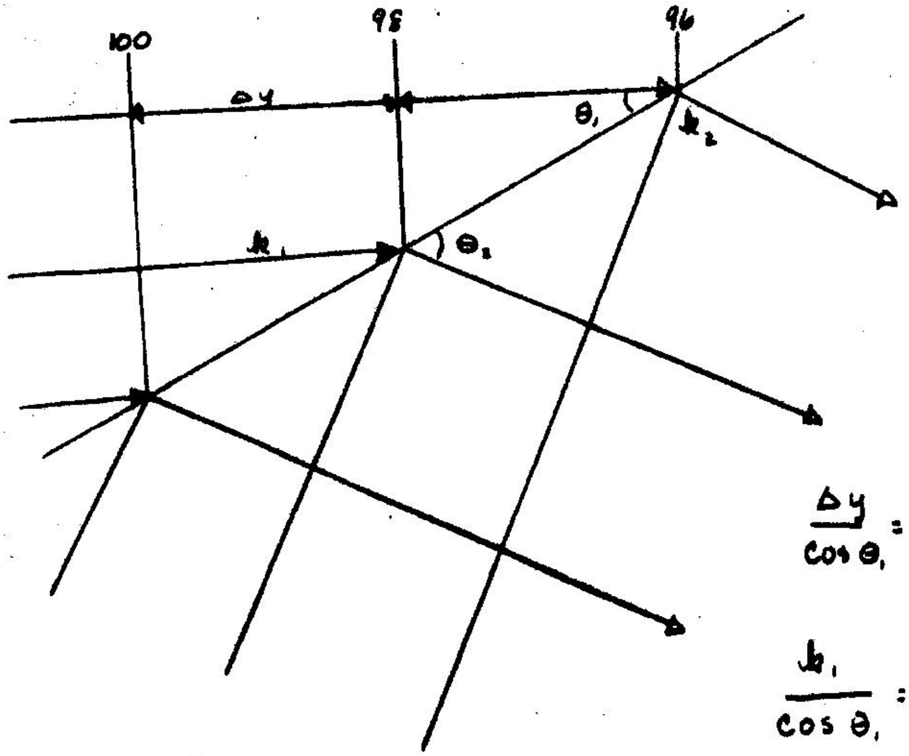
Transform the dimension so that

$$\bar{x} = \frac{x}{\sqrt{k_x}}, \quad \bar{y} = \frac{y}{\sqrt{k_y}}, \quad \bar{z} = \frac{z}{\sqrt{k_z}}$$

then

$$\frac{\partial^2 h}{\partial \bar{x}^2} + \frac{\partial^2 h}{\partial \bar{y}^2} + \frac{\partial^2 h}{\partial \bar{z}^2} = 0$$

Change of Media



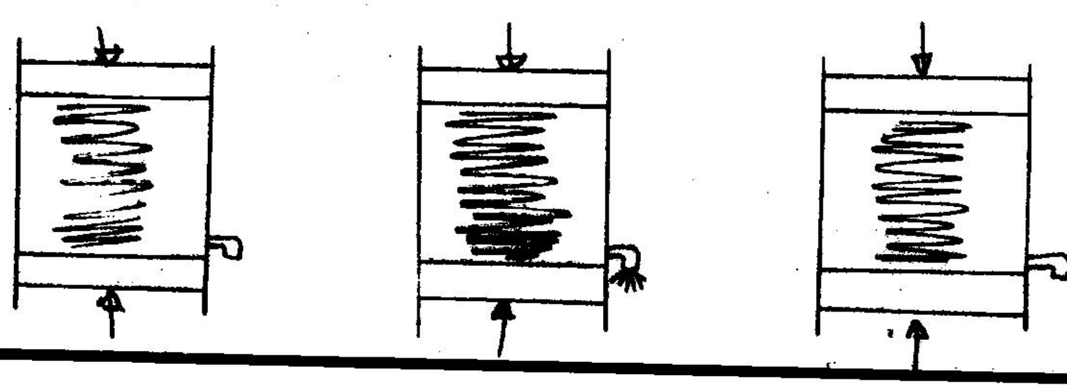
$$\frac{\Delta y}{\cos\theta_1} = \frac{\Delta y \, k_2/k_1}{\cos\theta_2}$$

$$\frac{k_1}{\cos\theta_1} = \frac{k_2}{\cos\theta_2} \quad \text{SNELL'S LAW}$$

Contours in K_2 region must be closer together all other things remaining the same. Smaller K means greater resistance to flow. But the equipotential line must match at the contact. Hence, a refraction of flow is required.

Consolidation Theory - Incompressible Fluid, Compressible Media

Consider the soil to be a compressible material. The relationship between the stress level and the voids is of interest here since in order to change the voids when they are filled with a fluid the fluid must be pushed out. The model will be that of a spring inside a container supporting two plungers, the container being filled with water. With the drainage cock turned off the water



will carry a portion of the load until it reaches equilibrium with the load; it then carries all the load, the water carrying none.

We need three things: the constitutive equation for the solid phase relating voids to pressure, the rate of flow as a function of pressure (Darcy's Law) and the continuity equation.

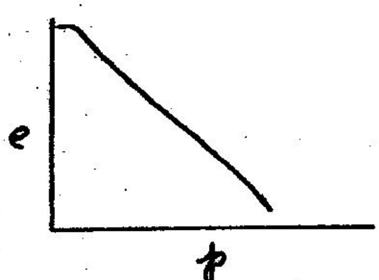
<u>The Consolidation Test</u>

A block of soil is tested by applying a sudden load and recording the settlement vs time until the settlement has virtually stopped. Two things are determined from such a test. The e vs p curves for final settlement are determined from the end points of the settlement for each application of load, but also the time delay in deflection gives data from which the effective permeability or consolidation coefficient may be determined experimentally. This latter is important since permeability in clay is so small as to make other methods for determination impractical.

For clay it appears that after reaching the original state of stress the deflection (or volume change) is an exponential function of p, namely

$$e = c \exp(p)$$

On semi-log paper this plots as a straight line



Over a small range of pressure change we may linearize $\frac{\partial e}{\partial p}$ to be

$$\frac{\partial e}{\partial p} = c_v$$

The rate of flow is given by Darcy's Law

$$Q = kiA$$

Assuming only flow up or down

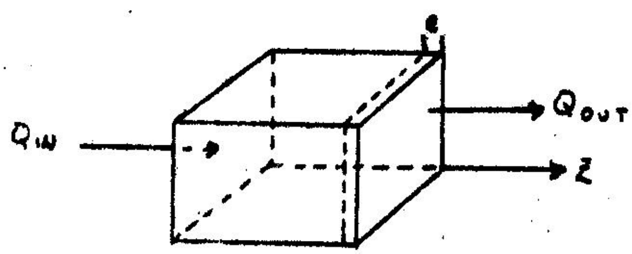
$$Q_{out} - Q_{in} = \frac{\partial e}{\partial t}$$

$$i = \frac{\partial h_{xs}}{\partial t} \quad ; \quad h_{xs}$$ = head in excess of steady state flow or equilibrium

<u>Total Settlement</u>

$$\delta = \int_0^H \frac{C_v p}{A} dx$$

The settlement progresses with the water in the space between the solid, being pushed out, governed by Darcy's Law:



$$Q_{in} = k i \, \Delta x \, \Delta y$$

$$Q_{out} = k\left(i + \frac{\partial i}{\partial z} \Delta z\right) \Delta x \Delta y$$

$$\frac{\partial e}{\partial t} \Delta z \Delta y \Delta x = Q_{out} - Q_{in}$$

$$= k \frac{\partial^2 h_{xs}}{\partial z^2} \Delta z \Delta x \Delta y$$

$$\partial e = C_v \partial p = \gamma_o C_v \, \partial h_{xs}$$

$$\gamma_o C_v \frac{\partial h_{xs}}{\partial t} = k \frac{\partial^2 h_{xs}}{\partial t^2}$$

or

$$\frac{\partial h_{xs}}{\partial t} = \frac{k}{\gamma_o C_v} \frac{\partial^2 h_{xs}}{\partial t^2}$$ the diffusion equation

This can be generalized to three dimensions

$$\frac{\partial h_{xs}}{\partial t} = \frac{k}{\gamma_0 C_v} \nabla^2 h_{xs}$$

STRUCTURAL DYNAMICS

Introduction

The use of orthogonal functions in solving dynamic problems in structures is not new. Engineering literature is well populated with articles and books giving detailed discussions and examples of the method. However, as yet, civil engineers have not adopted the procedure in the analysis of buildings under earthquake and blast loading. In the case of earthquake loading, the most common procedure is to design for an assumed horizontally applied load equal to some percentage of the weight of the structure. This percentage of the gravity force has been chosen on the basis of experience and judgement of practicing engineers. Blast design, being of general interest only recently due to its application in civil defense, has been treated more fully from the theoretical standpoint than has aseismic design.

However, as will be pointed out in this section, the theory and solutions obtained from blast problems are equally applicable to earthquake problems. Conversely, methods used in the analysis of the response of structures to ground motion, may readily be used to analyze the response of structures to blast. A great deal may be gained by comparing progress made in both fields.

The purpose of this section is to present the normal mode method of analysis in a simplified form. It is hoped that the simplified equations involving parameters having some physical significance will clarify the approach and make it more amenable to use in general design.

Nomenclature

Certain basic quantities used in the derivation to follow are shown in Figure 1. Other symbols will be defined as they are first used.

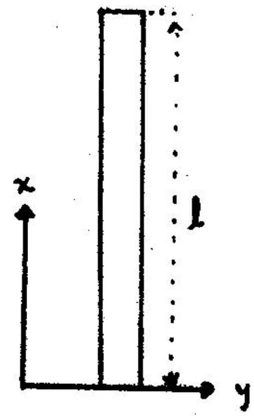


Fig. 1

E = Modulus of Elasticity

EI = Flexural Rigidity

e = Base of Natural Logarithms (2.73)

g = Acceleration Due to Gravity

I = Moment of Inertia

l = Length of the Beam

m = Mass Per Unit Length

M = Bending Moment

P = Force

Q = Generalized Force

q = Generalized Coordinate

t = Time

T = Kinetic Energy

U = Potential Energy

V = Shear

x = Distance Along the Beam

y,y,y = Displacement, Velocity, Acceleration

Theory

For the purpose of presentation, examples will be limited to cases involving structures which may be approximated by uniform beams. The differential equation of motion of a beam neglecting shear deformation and rotary inertia, has been derived many times. Two of the more convenient references are Timoshenko[1*] and Karman and Biot[2]. This differential equation may be solved by any one of the classical methods. It is known that these methods lead to a unique solution in terms of an infinite series of characteristic functions (normal modes) which are orthogonal. Much labor can hence be saved by adopting the common procedure of assuming the form of the solution and evaluating the coefficients of each term (method of normal coordinates).

The case of a beam subject to motion at the supports will be treated here, and similarities between this and other types of loading will be pointed out. No restrictions will be made as to end conditions for the beam, however, it will be postulated that all supports will undergo the same displacements. Expanding the deflection of the beam relative to its supports in terms of generalized or normal coordinates, we obtain,

$$y_{(x,t)} = y_o(x,t) + \sum_{i=1}^{\infty} \phi_{i(x)}\, q_{i(t)} \quad \ldots \qquad (1)$$

where

$y_o(x,t)$ = motion of the supports

$\phi_i(x,t)$ = characteristic function

$q_i(t)$ = generalized coordinate

The equations of motion of a beam in terms of its generalized coordinates are Lagrange's equations[2].

---

*Superscripts refer to references given at the end of the section.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \quad (i = 1, 2, 3, \ldots \infty) \tag{2A}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}_0}\right) - \frac{\partial T}{\partial y_0} + \frac{\partial U}{\partial y_0} = Q_0 \tag{2B}$$

where

$$U = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx \qquad \text{Potential Energy} \tag{3}$$

$$T = \frac{m}{2}\int_0^l \dot{y}^2\, dx \qquad \text{Kinetic Energy} \tag{4}$$

$$\left.\begin{aligned} Q_i &= \int_0^l P(x,t)\, \partial y/\partial q_i \, dx \\ Q_0 &= \int_0^l P(x,t)\, \partial y/\partial y_0 \, dx \end{aligned}\right\} \qquad \text{Generalized force} \tag{5}$$

Substituting Equation (1) in Equations (3) and (4) results in

$$U = \frac{EI}{2}\sum_{i=1}^{\infty} q_i^2 \int_0^l \left(\frac{d^2\phi_i(x)}{dx^2}\right)^2 dx \tag{6}$$

$$T = \frac{m \dot{y}_0^2 l}{2} + m\dot{y}_0 \sum_{i=1}^{\infty} \dot{q}_i \int_0^l \phi_i(x)\, dx + \frac{ml}{2}\sum_{i=1}^{\infty} \dot{q}_i^2 \tag{7}$$

In the case being considered the only force applied to the beam is applied to the supports. However, the motion of the supports is not a function of q_i. Therefore

$$Q_i = \int_0^l P(x,t)\frac{\partial y_0}{\partial q_i}\, dx = 0 \tag{8}$$

Substituting Equations (6), (7) and (8) into Lagrangian equations are obtained:

$$m\ddot{y}_0 \int_0^l \phi_i(x)\, dx + ml\, \ddot{q}_i + EI q_i \int_0^l \left(\frac{d^2\phi(x)}{dx^2}\right)^2 dx = 0 \tag{9}$$

The characteristic function represents the shape of the harmonically vibrating beam in the absence of external excitation. This function has been established as

$$\phi_i(x) = A\sin\beta x + B\cos\beta x + C\sinh\beta x + D\cosh\beta \ldots \quad (10)$$

where $\beta^4 = \frac{m\omega^2}{EI}$. This may be verified by study of reference number 1. The arbitrary constants are determined by the boundary conditions imposed on the beam and the condition of orthogonality, namely

$$\int_0^l m\phi_j\phi_i(x)\,dx = \begin{matrix} \text{zero for } i \neq j \\ \text{Arbitrary for } i = j \end{matrix} \quad (11)$$

For the case i=j, it is convenient to normalize the equation by letting the arbitrary value of the integral be equal to ml. It follows then that

$$\int_0^l m\phi_i^2(x)\,dx = ml \quad (12)$$

Defining ϕ'' as $\frac{1}{\beta^2}\frac{d^2\phi(x)}{dx^2}$ it follows from Equations (10) and (12) that Equation (9) becomes,

$$m\ddot{y}_0\int_0^l \phi_{i(x)}\,dx + ml\ddot{q}_i + EIl\beta_i^4 q_i = 0 \quad (13)$$

Rearranging and defining $\frac{EI}{m}\beta_i^4 = \omega_i^2$

$$\ddot{q}_{i(t)} + \omega^2 q_{i(t)} = -\ddot{y}\frac{1}{l}\int_0^l \phi_{i(x)}\,dx \quad (14)$$

It can be shown[1] that w_i are natural undamped frequencies of free vibration and that for conditions of zero velocity and acceleration at time equal zero, Equation (14) has the following general solution:

$$\ddot{q}_{i(t)} = -\frac{1}{\omega_i}\int_0^t \ddot{y}_0(\eta)\frac{1}{l}\int_0^l \phi_{i(x)}\,dx \cdot \sin\omega_i(t-\eta)\,d\eta \quad (15)$$

$$q_i(t) = -\frac{1}{\omega_i^2}\frac{1}{\ell}\int_0^{\ell}\phi_i(x)\,dx \cdot \omega_i\int_0^{t}\ddot{y}_0(\eta)\sin(t-\eta)\,d\eta \tag{16}$$

The above equations solve for the unknown function q_i, and theoretically the problem is solved. If the same procedures were followed for the case of an arbitrary impulsive load on a beam, the following solution for q_i would be derived:

$$q_i(t) = \frac{P_0}{\omega_i^2 \ell}\cdot\frac{1}{\ell}\int_0^{\ell}\phi_i(x)\,\bar{w}(x)\,dx \cdot \omega_i\int_0^{t} f(\eta)\sin\omega_i(t-\eta)\,d\eta \tag{17}$$

where the applied load $w(x,t) =$

$$\frac{P_0}{\ell}\,\bar{w}(x)\,f(t) \tag{18}$$

P_0 ... Maximum total blast force.

$\bar{w}(x)$ ... Space distribution of force.

$f(t)$ ... Time variation in the force.

In the above solution for blast loading it has been assumed that the load may be broken down in the manner indicated. If for any reason the space distribution of force is a function of time or the time variation of force is a function of distance the above solution is invalid. However, many times blast loadings may be approximated closely by the above assumption.

Simplified Equations

In fairly recent years studies have been made which simplify the numerical work involved in using the normal mode approach. One of the most important publications has been the tabulation of characteristic numbers, characteristic functions and the successive derivatives of the characteristic functions. These tables are due to Young and Felgar and have been published by the

University of Texas.[3] Other investigators, such as G. W. Housner* and W. T. Thomson** have applied electronic computers in solving the time varying part of the solution.[4,5]

In order to see how these various investigations fit together, it is convenient to express Equations (16) and (17) in the following form:

$$q(t) = \frac{C k_i D_i}{(\beta l)^4} \tag{19}$$

where,

For Blast

$$C = \frac{P_0 l^3}{EI}$$

$$k = \frac{1}{l}\int_0^l \phi_i(x)\,\overline{W}(x)\,dx$$

$$D_i = \omega_i \int_0^t f(\eta)\sin(t-\eta)\,d\eta$$

$$(\beta l)^4 = \frac{\omega_i^2 m l^4}{EI}$$

For Earthquake

$$C = \frac{m g l^4}{EI}$$

$$k_i = \frac{1}{l}\int_0^l \phi_i(x)\,dx$$

$$D_i = \omega_i \int_0^t \frac{\ddot{y}_0}{g}\sin\omega_i(t-\eta)\,d\eta$$

$$(\beta l)^4 = \frac{\omega_i^2 m l^4}{EI}$$

With the exception of the factor C, all of the above coefficients are non-dimensional. By comparison between the two columns it is possible to see the similarities as well as the essential differences between earthquake and blast response.

Participation Factor

The coefficient k_i determines to what extent various modes participate in the statical deflection for the blast case. It is, therefore, called the participation factor. The value this factor takes is a weighted average of the characteristic function $\phi_i(x)$. When the exciting force is uniformly distributed over the length of the beam, $\overline{W}(x) = 1$, and the participation

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** Professor of Engineering, University of California.

factor for blast becomes a straight average of $\phi_i(x)$.

As may be seen from the coefficient C, the weight of the structure is analogous to an exciting force, in the earthquake case. In as much as a uniform beam is under consideration, it is consistent that k_i for the earthquake case is shown as a straight average of $\phi_i(x)$. If the beam had a distribution of weight $\overline{W}(x)$, the k_i for earthquake would be the same as for blast.

Dynamic Load Factor

A dynamic load factor is so termed because it tells how much the static response is magnified to obtain the dynamic response. This is precisely what the factor D_i does. For the blast case the integral involved the time variation of load $f_{(t)}$ while in the earthquake case the time variation in acceleration (expressed as percentage of gravity) is substituted. It should be noted that the dynamic load factor must be applied to the response of each mode separately, since in general D_i will be different for different modes.

For design purposes it is the maximum value of D_i that is of interest. Although there is no certainty that the maximum values of D_i will occur at the same time for various modes, the times are not greatly different and one must always consider the worst probable condition in design. The determination of the dynamic load factor for all but the most simple load histories becomes a very difficult numerical task. It is here that analog computers have come to the aid of the engineer.

G. W. Housner observed that the quantity D_i represented the acceleration in percent of gravity of a one degree of freedom structure of frequency W_i. Substituting real earthquake accelerograms for the term $\ddot{y}_o$ and determining the maximum value of the integral for all reasonable values of frequency, a distribution of acceleration versus period could be drawn which is similar to a distribution of light intensity versus wave length. For this reason the curves of maximum D_i versus period which have been obtained by means of electronic computers have been referred to as earthquake spectra.(4)

Damping may be included in the differential equation of motion of a beam by including a force term which is proportional to velocity and in phase with the deflection. This results in the addition of an exponential term in the dynamic load factor, namely

$$D_i = \omega_i \int_0^t f(\eta)\, e^{n\omega_i (t-\eta)} \sin \omega_i (t-\eta)\, d\eta \tag{20}$$

where n is the per unit of critical damping. Housner has shown the effect of the inclusion of damping on the earthquake spectra.

Most of the published work giving actual values of maximum dynamic load factors has been for earthquake excitation. However, the analog computer techniques are equally applicable to blast excitation. W. T. Thomson has been carrying on research at the University of California along these lines.

Characteristic Values

There are two characteristic values that enter in the solution which are properties of the freely vibrating beam. One of these values is the characteristic function $\phi_i(x)$, which describes the shape of the ith mode. The other value is the characteristic number $\beta_i \ell$. For uniform beams the characteristic values are determined by end conditions and Equations (11) and (12).

Young and Felgar[3] have tabulated values of $\phi_i(x)$, $\phi_i'(x)$, $\phi_i''(x)$, $\phi_i'''(x)$ and $\beta_i \ell$ for all combinations of clamped, pinned and free end conditions with the exception of the pinned-pinned case which results in a simple sine function. The primed values are the successive derivatives of $\phi_i(x)$ divided by successive powers of β_i in order to retain non-dimensionality.

Moment and Shear

Substituting the factors described above into Equation (1):

$$y_{(x,t)} = y_o(t) + \frac{mg\ell^4}{EI} \sum_{i=1}^{\infty} \frac{\phi_i(x)\, k_i(x)}{(\beta\ell)_i^4} D_i(t) \tag{21}$$

For the blast solution

$$y_{(x,t)} = \frac{P_o\ell}{EI} \sum_{i=1}^{\infty} \frac{\phi_i(x)\, k_i(x)}{(\beta\ell)_i^4} D_i(t) \tag{22}$$

The designing engineer is usually more interested in moments and shear than in deflection. Since $\phi_i(x)$ is the only variable quantity in the equation above that is a function of x, it is the only value that changes.

For earthquake:

$$M_{(x,t)} = mg\ell^2 \sum_{i=1}^{\infty} \frac{\phi_i''(x)\, k_i(x)}{(\beta_i\ell)^2} D_i(t) \tag{23}$$

For blast:

$$M_{(x,t)} = P_o\ell \sum_{i=1}^{\infty} \frac{\phi_i''(x)\, k_i(x)}{(\beta_i\ell)^2} D_i(t) \tag{24}$$

For earthquake:

$$V_{(x,t)} = mg\ell \sum_{i=0}^{\infty} \frac{\phi_i'''(x)\, k_i(x)}{\beta_i\ell} D_i(t) \tag{25}$$

For blast:

$$V_{(x,t)} = P_o \sum_{i=0}^{\infty} \frac{\phi_i'''(x)\, k_i(x)}{\beta_i\ell} D_i(t) \tag{26}$$

It should be noted that these solutions have neglected shear deformation and rotatory inertia. However, for most engineering purposes these effects are negligible.

Application

Earthquake

The Southern California earthquake of July 21, 1952 resulted in widespread damage in the region of Arvin, Tehachapi and Bakersfield, California. Much of this damage was due to the lack of resistance to lateral forces. Some newer structures, however, had been designed under the earthquake provisions of modern building codes. The performance of this latter group of structures is a true test of the validity of design techniques. As an example of the normal mode method of analysis, the Arvin High School Administration Building will be studied.

The Arvin High School Administration Building is a two story reinforced concrete building with brick veneer except that the second story wall at the west end was 8 1/2 inch reinforced grouted brick masonry without openings.

Following is a description of the damage:[6]

The west end second story wall was fractured. Subsequent shocks increased the damage and the wall became badly cracked. The adjoining concrete wall on the north wall (faced with brick veneer) cracked along a horizontal construction joint. After the west wall cracked, later shocks caused plaster damage and spalls to metal lath and plaster cross partitions which attempted to resist forces for which they were not designed, dependence in design having been placed on the exterior west wall. There was also evidence that the second story wall at the south rocked somewhat because there were minor brick spalls at their base. There was no collapse.

Investigators studying the cause of the damage stated that the wall was of faulty construction although there was reason to believe that the flexibility of the 200 foot long roof diaphragm contributed to the failure.

The plot plan and section of the Administration Building are shown in Figure 2. The loads entering the end wall during an earthquake may be approximated roughly by analyzing a uniform beam of similar dimensions to the roof slab. The moment of inertia is approximately 90x10^6 inches^4.

The force entering the end wall may be computed using Equation 25,

$$V = mg\,\ell \sum_{i=0}^{\infty} \frac{\phi'''_{i(x)}\, k_i\, D_{i(t)}}{\beta_i \ell} \qquad (25)$$

It shall also be assumed that the interior columns and partitions give vertical support to the diaphragm, but do not offer resistance to lateral motion. The diaphragm may be considered to be a beam with end conditions lying somewhere between full fixity and simple support. The condition of simple support will be assumed first.

The characteristic function for a simply supported beam is given by

$$\phi_{i(x)} = \sqrt{2}\ \sin \frac{i\pi x}{\ell} \qquad (27)$$

The characteristic number is

$$\beta_i \ell = i\pi \qquad (28)$$

The following is a tabulation of the computed dynamic parameters:

i	1	2	3	4
$\phi_{i(x=0)}$	0	0	0	0
$\phi'''_{i(x=0)}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
$(\beta_i \ell)$	π	2π	3π	4π
k_i	$\frac{2\sqrt{2}}{\pi}$	0	$\frac{2\sqrt{2}}{3\pi}$	0
$\sqrt{\frac{EI}{m\ell^4}}\ T_i$	$\frac{2}{\pi}$	$\frac{1}{2\pi}$	$\frac{2}{9\pi}$	$\frac{1}{8\pi}$

The fundamental period of vibration is 0.34 seconds wherein the following values were assumed:

$$E = 2.5 \times 10^6$$

$$I = 90 \times 10^6 \text{ inches}^4$$

$$m = 114 \frac{\text{lb.-sec}^2}{\text{ft}^2} = .79 \frac{\text{lb.-sec}^2}{\text{in}^2}$$

$$\ell = 2400 \text{ inches}$$

Since there is no spectrum available for Arvin in the shock of July 21, 1952, the spectrum for El Centro in the shock of May 18, 1940, component N-S will be used. See Figure 3A. Therefore, assuming a damping factor of 0.10 we may pick the dynamic load factor off the spectrum. This yields a value of approximately

$$D_i = 0.8$$

Since the fundamental period of vibration is by definition the longest period excitable, all other values of D_i may be taken as 0.8.

Substituting into Equation (25) gives the following results:

$$V = mg\ell \left\{ \frac{\phi_1''' K_1 D_1}{\beta_1 \ell} + \frac{\phi_2''' K_2 D_2}{\beta_2 \ell} + \frac{\phi_3''' K_3 D_3}{\beta_3 \ell} + \dots \right\}$$

$$= 114 \times 32.2 \times 200 \left\{ \frac{\sqrt{2}\,(2\sqrt{2}/\pi)\,0.8)}{\pi} + 0 + \frac{\sqrt{2}\,(\frac{2\sqrt{2}}{3\pi})\,0.8}{3\pi} \right\}$$

$$= 265{,}000 \,\#$$

The area of the wall cross section is approximately 3000 square inches. Therefore, the average shearing stress on the end wall is 89 psi, which could cause failure. However, the partitions should help to reduce this stress and the dynamic load factor at Arvin may have been less than at El Centro in 1940.

If in the above analysis end conditions of full fixity were assumed the same answer would result since increased restraint tends to decrease the period of vibration.

A very important point has been demonstrated here. In the above example the solution could just as readily have been obtained by multiplying the static shear at the wall under an mg loading by the dynamic load factor D_i. This follows from the definition of D_i i.e., the factor by which the static response is multiplied to obtain the dynamic response. Structures having long periods of vibration take advantage of the reduction in the value of D_i as shown on the spectrum.

Blast

A somewhat more hypothetical example for blast type loading could be obtained by analyzing the above structural element under an impulsive loading. If we assume a pulse type load of 0.3 seconds duration time, the maximum D_i plotted against period takes the appearance shown in Figure 3B. This curve might reasonably be called the blast spectrum.

Since the period of the roof slab is 0.34 seconds, the dynamic load factor for all modes is roughly 2.0. In other words, the dynamic response is twice the static response under the maximum blast load.

CONCLUSION

In the foregoing discussion it has been shown how the response of beams under dynamic loads may be expressed as a series expansion involving four non-dimensional parameters and one dimensional constant. The equation takes the following form:

$$y = C \sum_{i=0}^{\infty} \frac{\phi_i(x)\, k_i\, D_i(t)}{(\beta_i l)^4} \qquad (29)$$

This form of solution is not limited to beams. In fact all structures subject to dynamic excitation, where the principle of superposition is valid and loads may be broken up into a time variation times a space distribution, may be analyzed using the above form of solution.

Prototype structures must be approximated by simplified models in order to satisfy the above conditions. Structures having uniform distributions of mass such as stacks, silos or individual structural elements may be approximated by beam theory. Buildings where weight is concentrated at floor levels may be approximated by the so called shear building. Structures having large shearing deflections in comparison with their bending deflections may be approximated by the so called shear beam.

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4. Housner, Martel and Alford, Spectrum Analysis of Strong-Motion Earthquakes, Bulletin of Seismological Society of America, Volume 43, No. 2, April 1953.
5. Thomson, W. T., Impulsive Response of Beams in the Elastic and Plastic Range, Journal Applied Mechanics, Sept. 1954.
6. Steinbrugge and Moran, An Engineering Study of the Southern California Earthquake of July 21, 1952, Bull. of Seis. Soc. of Am. Vol. 44, No. 2B, April 1954.

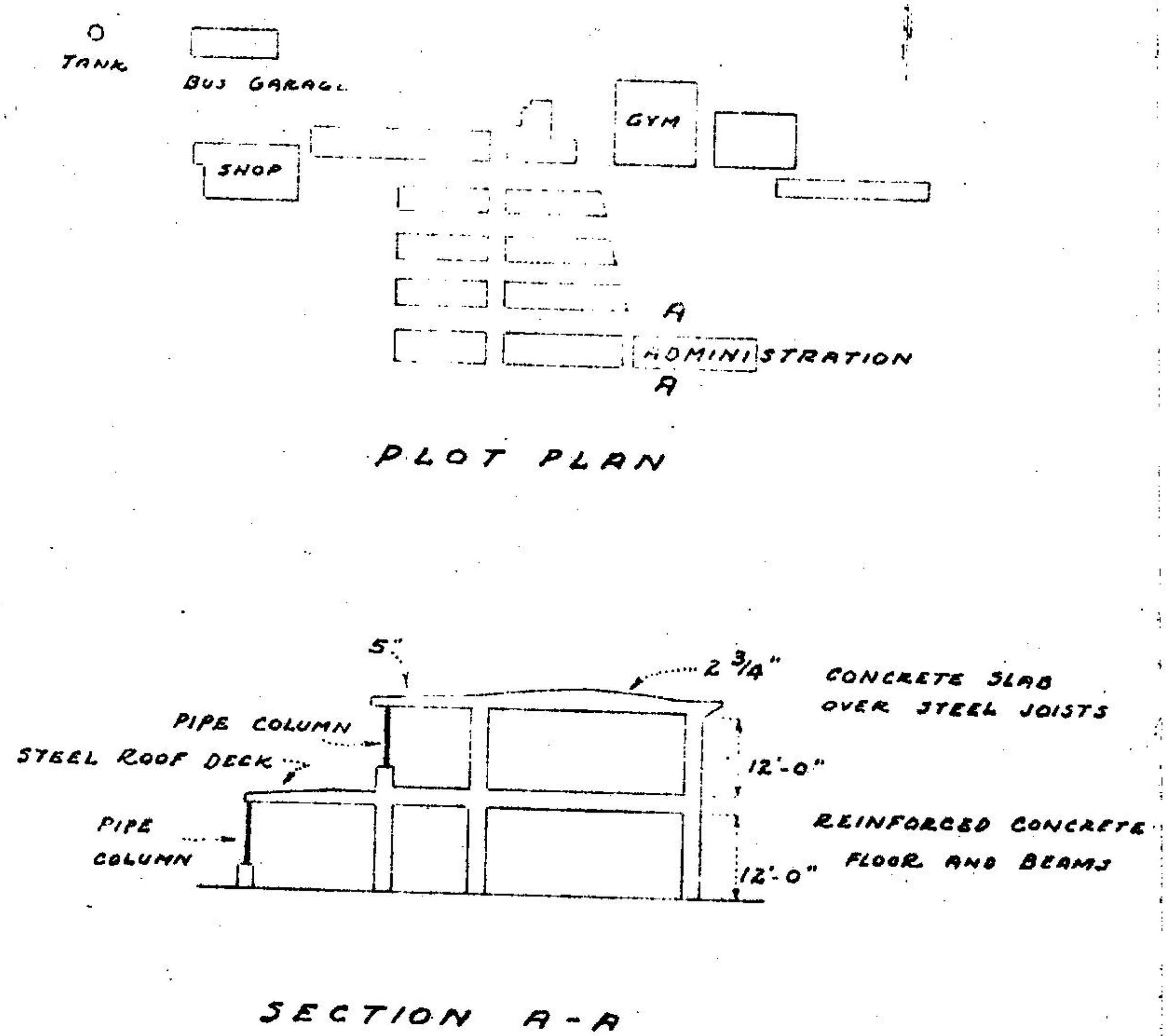


Figure 2 - Arvin High School

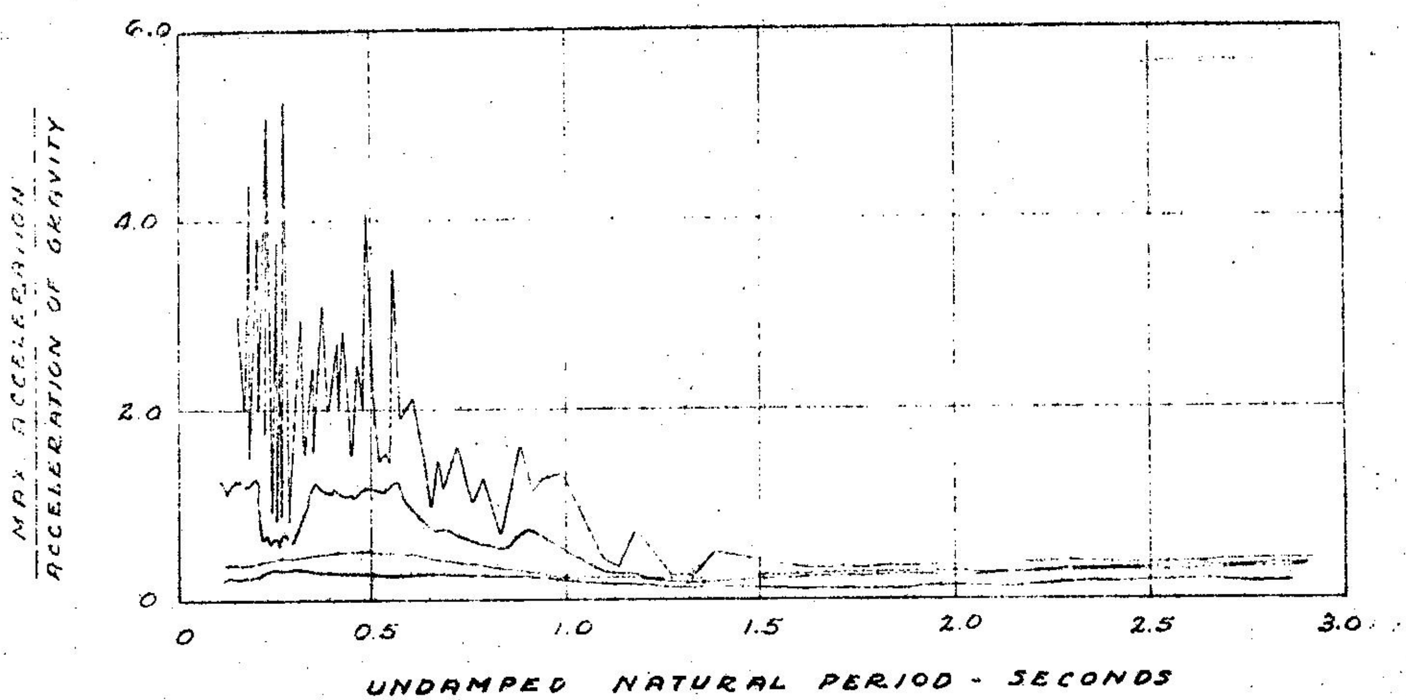


Figure 3A - Acceleration spectrum for El Centro, California Earthquake of 18 May 1940, Component N-S

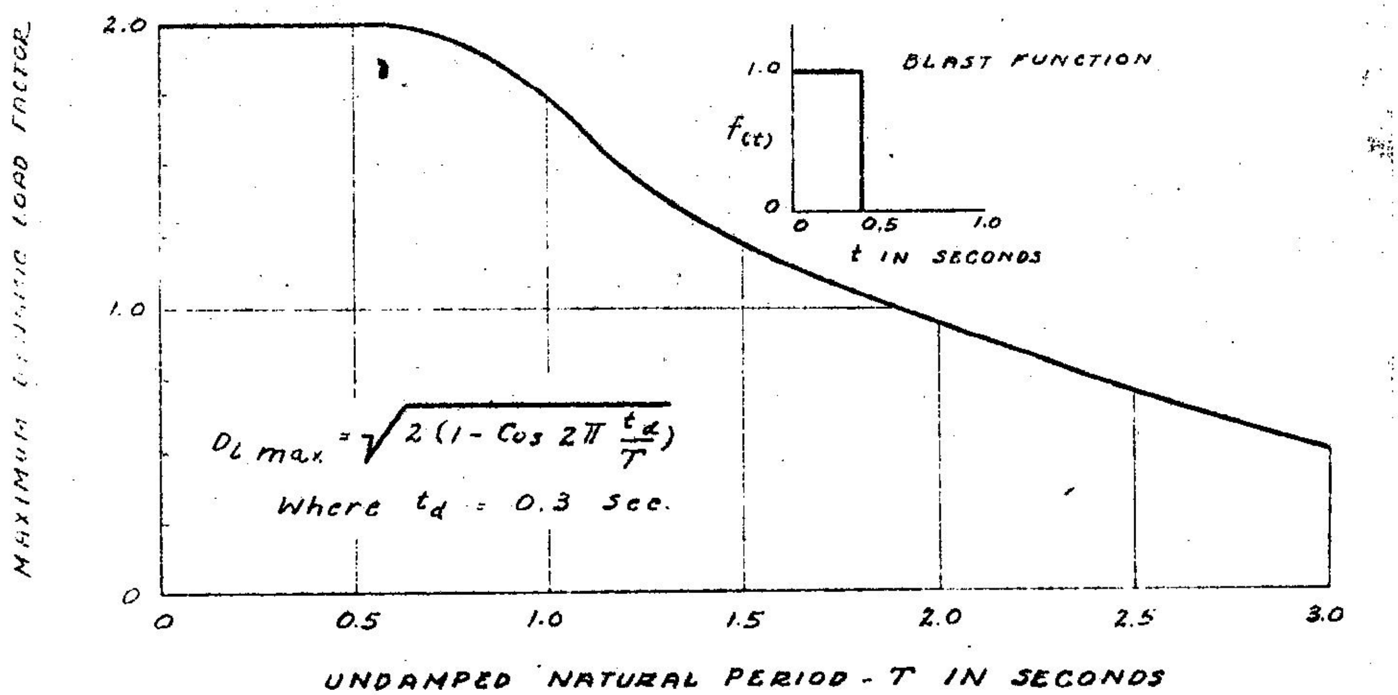


Figure 3B - Maximum dynamic load factor for step function shock of duration 0.3 seconds versus undamped natural period - T.

LARGE-DIAMETER DRILLING FOR EMPLACEMENT OF NUCLEAR DEVICES

Ref: Proc. of Third Plowshare Symposium p. 239-268

Drilling Methods	Diameter	Depth
1. Churn Drilling	4'	600-900'

30" requires one pass

30" more than one pass

Penetration rates 28"Ø 12'/hr to < 6"/hr

A percussion type drill consisting of a long steel bit that is mechanically lifted and dropped to disintegrate the rock.

2. Auger Drilling	15'	65'

Useful in soft material that can stand open during drilling. However, drillers mud or casing can be used in softer materials 36" hole 100' 24 hours.

3. Core Drilling	8'	200'

Chilled steel shot is used as cutting material.

See p. 252 Peurifoy, Construction Planning, Equipment and Methods

4. Rotary Drilling	130"	570'
	90"	1400'
	25"	1700'

Like oil well drilling.

Cost

1. Churn Drilling — $20/ft - $40/ft
2. Auger — $ 1/ft for 36"Ø soft; $20/ft for 72"Ø hard
3. Core Drilling — $12/ft - $120/ft soft to hard for 36" hole
4. Rotary Drilling

Depth/Bore Diameter

Casing ID	100-500	500-1000	1000-1500	1500-2000	2000-2500
36	44	48	50	52	54
48	56	62	64	66	70
60	72	74	78	80	84
66	78	80	84	88	90

Prof. Talley will talk on device size Monday hence for now we will consider the following dimensions:

Diameter	Depth
12"ID	100'
36	750
66"	2500'

Example 36" @ 750'

See: Third Plowshare Conf. Proceedings p. 239-268

Actual boring will be 48' in diameter and will be drilled by a drill rig with 700 input horsepower. The drill rig cost will be $1200 per day operating and $800 per day non-operating.

1. Rig Mobilization

a. Hauling 1000 miles @ $10/mile		$10,000
b. Rig cost - 250 miles/day, 4 days @ 800.00		3,200
	Total	$13,200

2. Site Preparation

32 dozer hours @ $15.00	480

3. Rig-up

a. Rig time 3 1/2 days @ $1200-		$4,200
b. Hanler support		1,200
	Sub Total	$5,400

4. Surface Hole - 50' deep

a. Drilling 54" hole

(1) Rig time - 15 hrs @ $50.00	$750
(2) Cuttercosts - 50' @ $21.00	1,050
(3) Fluid costs	---
b. Surface Pipe	
(1) Pipe - 6696 lbs @ $0.20	1,339
(2) Cement - 115 ft^3 @ $1.50	172
(3) Rig time - 2 days @ 1200	2,400
Sub Total	$5,711
5. Drill 48" hole 50' - 750	
a. Rig time 200 hours @ $50.00	$10,000
b. Cutter costs - 700 @ $15.00	11,200
c. Fluid Costs - 2500 Bbls @ $2.00	5,000
	$26,200
6A. Condition hole and run heavy casing	
a. Casing Cost - 750' of 3/4" wall 226,500 lbs @ $0.20	$45,000
b. Casing welding cost $36 joint for 19 joints	684
c. Cement cost 3700 ft^3 @ $1.50	5,050
d. Cement pumping	400
e. Rig time - 8 days @ $1,200	9,600
Sub Total	$60,834
6B. Run light perforated casing	
a. Casing cost 750' @ $10.50	$7,875
b. Rig time 2 days @ $1,200	2,400
Sub Total	$10,275

7. Tear Down

a. Rig time, 3 days @ $1,200		3,600
b. Hanler support		1,000
	Sub Total	$4,600

8. Rig demobilization 13,200

9. Total Costs

a. Heavy Casing	129,565
or	$172/ft
b. Light Casing	79,006
or	$105/ft

MATHEMATICAL INSTABILITIES

Because practical problems in hydrodynamics are usually describable in terms of non-linear models and because the physical situations are two or three dimensional, analytic solutions are not to be found. Numerical analysis can produce some answers, but even this technique fails at times. For instance, the two dimensional pressure profile of air circulation about an infinitely long wing can be obtained to any required accuracy by use of a digital computer. But when the wing is finite, the problem becomes three dimensional and is beyond the capabilities of present day high speed digital computers.

Even the relatively simple problem of one-dimensional time dependent flow, which actually can be handled by computers, requires some special care. Thus numerical analysis can introduce instabilities which have nothing to do with the physics of the situation. We can best illustrate the point by outlining the numerical solution of a particular partial differential equation, the hyperbolic equation:

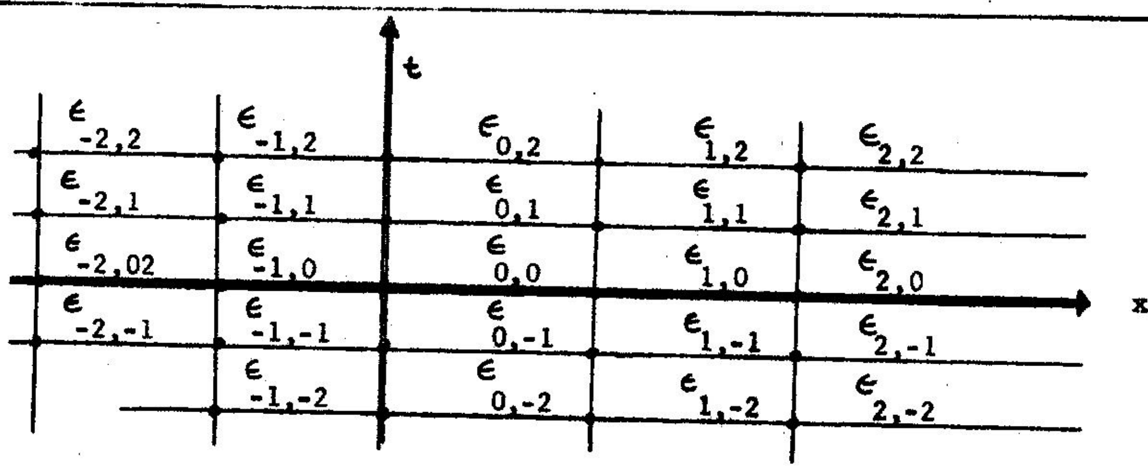
$$\frac{\partial^2}{\partial t^2} u(x,t) - c^2 \frac{\partial^2}{\partial x^2} u(x,t) = 0 \qquad (1)$$

which is the simplest general equation describing hydrodynamic flow. In the general case, c itself depends on the derivatives of the displacement u and thus (1) becomes non-linear. Let us suppose that we try a solution to (1) of the form u + ϵ , where u satisfies (1) exactly and ϵ is a small additional term, introduced, say, by rounding-off errors. Substitution of the trial solution u + ϵ into (1) produces

$$\frac{\partial^2}{\partial t^2} \epsilon(x,t) - c^2 \frac{\partial^2}{\partial x^2} \epsilon(x,t) = 0 \tag{2}$$

where c is known from the solution of (1). Since ϵ is small, the dependence of c on ϵ may be neglected and (2) can be treated as a linear equation. We are concerned with the behavior of this error, ϵ , as x and t change.

The numerical differencing technique used to deal with (2) is pictured in the Figure: a lattice is constructed such that as we move upward we are moving forward in time in increments of Δt, and if we move to the right or left we move in the positive or negative x direction in increments of Δx. Note the notation used: $\epsilon_{m,n}$ means the value of ϵ at $x = m\Delta x$ and $t = n\Delta t$; naturally, $\epsilon_{0,0} = \epsilon(0,0)$.



The lattice used for finding the behavior of ϵ by means of a difference equation.

Assume that ϵ is known at every point in space for all times up to t = 0. We can then move forward in time and calculate ϵ at any point. Actually, our assumption is unnecessarily strong: (2) is a second order equation in time, so it will suffice to know $\epsilon_{m,0}$ and $\epsilon_{m,-1}$ for all m, that is, for all x at t = 0 and t = $-\Delta t$. Let us study a general Fourier component of the error. Since (2) is linear the solutions can be obtained by the superposition of Fourier components. Thus we shall be able to predict the behavior of all possible error.

We consider

$$\epsilon_{m,0} = \epsilon_{0}(k)\, e^{jkm\Delta x} \tag{3}$$

and

$$\epsilon_{m,-1} = \epsilon_{-1}(k)\, e^{jkm\Delta x} \tag{4}$$

where $j = \sqrt{-1}$, k is a wave number, and ϵ_0 and ϵ_{-1} are arbitrary (but given) coefficients independent of space and time.

A difference equation can be formed from (2) by returning to the definition of the derivative[(1)]:

$$\left.\frac{\partial^2}{\partial t^2}\epsilon\right|_{\substack{x=0\\t=0}} = \frac{\dfrac{\epsilon_{0,1}-\epsilon_{0,0}}{\Delta t} - \dfrac{\epsilon_{0,0}-\epsilon_{0,-1}}{\Delta t}}{\Delta t} = \frac{\epsilon_{0,1} - 2\epsilon_{0,0} + \epsilon_{0,-1}}{(\Delta t)^2} \tag{5}$$

$$\left.c^2\frac{\partial^2}{\partial x^2}\epsilon\right|_{\substack{x=0\\t=0}} = c^2\,\frac{\dfrac{\epsilon_{1,0}-\epsilon_{0,0}}{\Delta x} - \dfrac{\epsilon_{0,0}-\epsilon_{-1,0}}{\Delta x}}{\Delta x} = c^2\,\frac{\epsilon_{1,0} - 2\epsilon_{0,0} + \epsilon_{-1,0}}{(\Delta x)^2} \tag{6}$$

$$= \frac{2c^2}{(\Delta x)^2}\,\epsilon_0(\cos k\Delta x - 1) \tag{7}$$

---

(1) Equations for x = 0 will be similar except for a common factor $e^{jkm\Delta x}$. Dividing by this factor, one is led back to (9).

which last follows from direct substitution of (3) into (6). Equating (5) and (7) produces the difference equation corresponding to the differential equation (2):

$$\epsilon_{0,1} - 2\epsilon_{0,0} + \epsilon_{0,-1} = \frac{2c^2}{\left(\frac{\Delta x}{\Delta t}\right)^2} \epsilon_0 \left[\cos k\Delta x - 1\right] \quad (8)$$

or using (3) and (4),

$$\epsilon_1 - 2\epsilon_0 + \epsilon_{-1} = \frac{2c^2}{\left(\frac{\Delta x}{\Delta t}\right)^2} \epsilon_0 \left[\cos k\Delta x - 1\right] \quad (9)$$

from which it follows that

$$\epsilon_1 = \frac{2c^2}{\left(\frac{\Delta x}{\Delta t}\right)^2} \epsilon_0 \left[\cos k\Delta x - 1\right] + 2\epsilon_0 - \epsilon_{-1} \quad (10)$$

Insertion of a word or two concerning the validity of (10) seems appropriate. As Δx and Δt approach dx and dt, the difference equation approaches the differential equation and becomes exact. For finite Δx and Δt, (10) is an approximation in which terms of second order in Δx and Δt have been neglected. Surprisingly, even for small Δx and Δt, we shall see that what matters in the behavior of the error term is the ratio of $\Delta x/\Delta t$ to c.

For $c^2 \ll \left(\frac{\Delta x}{\Delta t}\right)^2$ (10) becomes

$$\epsilon_1 = 2\epsilon_0 - \epsilon_{-1} + \ldots \quad \text{(small corrections).}$$

At each step forward in time (remember that the indices in (10) refer only to the time increments) the influence of the side points will be very weak. Each step represents a very small extrapolation into the future and the situation is similar to that of a total differential equation depending on time alone. This result appeals to common sense: while Δx and Δt must both be small, if $\Delta x \ll c\Delta t$, a signal from the side points cannot arrive at the point of interest until long after the time increment step has been completed.

For $c^2 \gg \left(\frac{\Delta x}{\Delta t}\right)^2$, (10) becomes

$$\epsilon_1 = \frac{2c^2}{\left(\frac{\Delta x}{\Delta t}\right)^2} \epsilon_0 \left[\cos k\Delta x - 1\right] + \text{(small corrections)} \qquad (12)$$

There will be no problem if there is not much spatial variation in the error. This corresponds to a small wave number, $k \sim 0$, a very long wave-length wave. Then $\cos k\Delta x = 1 - (k\Delta x)^2/2$. (12) for this approximation is

$$\epsilon_1 = \frac{-c^2}{\left(\frac{\Delta x}{\Delta t}\right)^2} \epsilon_0 (\Delta x)^2 k^2 \qquad (13)$$

While $c^2/\left(\frac{\Delta x}{\Delta t}\right)^2$ is large, $(\Delta xk)^2$ is small and $\epsilon_1 < \epsilon_0$; the error term begins to die out.

On the other hand, there will be some value of k such that $\cos k\Delta x = -1$: the value of ϵ changes sign as we move horizontally across the grid. (12) is then

$$\epsilon_1 = -4\frac{c^2}{\left(\frac{\Delta x}{\Delta t}\right)^2} \epsilon_0$$

Thus ϵ_1 is greater than ϵ_0 and the error grows exponentially as time increases.

Physically, what has happened is that before the temporal step is finished, signals reach the new point from points far to the sides. The calculation, which is limited to neighboring points, ignores this fact. There are at least two ways to cure the problem. The first is that since the space and time increments are such that "sonic" signals from more than just neighboring points reach the new point of interest, simply include the additional terms in (10) needed to account for the other signals. The drawback to this method is that the recursion relation becomes much more complicated. The second method is to alter the size of the lattice spacing so that $\Delta t < \Delta x/c$, which has the distinct advantage that it keeps the simple form of (10).

The situation just described is the well known Courant Instability. Other mathematical instabilities arise in the neighborhood of shocks and still others must be considered when two dimensional flow problems are to be solved. The electronic computer will become an even more important tool in hydrodynamics. But it can be useful only when handled with caution and mathematical insight. The use of computing machines requires ingenuity which is most un-machine-like.

APPENDIX B

PARTICIPATING FACULTY LECTURES

The notes to follow have been prepared by participating UPR Faculty Members:

L. E. Mora-Faría, Department of Civil Engineering

Canals

J. Ríos Carvajal, Department of Mechanical Engineering

Project Carryall (Highway and Railroad cuts)

N. Beylerian, Department of Civil Engineering

Project Chariot (Harbors)

D. Taylor, Department of Chemical Engineering

Project Gasbuggy (Gaswell stimulation)

F. Muñoz-Ribadeneira, Department of Chemical Engineering

Mining Applications

K. Pedersen, Department of Nuclear Engineering

Geothermal Heat and Salt Water Conversion

M. Saca, Department of Physics

Isotope Production

CANALS

L. Mora-Farfa

I. Interoceanic Sea Level Canal

A. On site surveys: Authorized to start January 1967 (U.S. Government)

B. Acoustic Wave Program: Rockets 100,000-200,000 ft (U.S. Government) metal chaff.-radar tracked

C. Bioenvironmental Program: Radiological Safety feasibility of using (B.M.I.) nuclear explosives

1. Marine physiochemistry and biology
2. Fish resources in estuaries and oceans on both sides of the isthmus
3. Terrestrial, freshwater, agricultural, and marine ecologies
4. Radiohydrology
5. Human ecology
6. Radiation dose estimation

D. Meteorology: Transportation and deposition of radioactivity

Studies: wind patterns and rainfall two weather stations

II. Excavation Research and Development

A. Special Explosives and Emplacement Techniques

B. Cratering Calculations

C. Cratering Experiments

III. The Transisthmian Canal

Problems: Emotional
Political
All economic consequences are predicted; complete safety is assured.

IV. Nuclear Explosives: Cheapest way of moving large quantities of earth.

Explosives

	Chemical	Nuclear
Size	Large	Small
Energy Density	Small	Large
Temperature	Low	Very High
Pressure	High	Very High

The largest conventional explosive is larger than the smallest nuclear explosive.

Prices: \$350,000 - 10 kt
\$600,000 - 2 Mt
Arming and firing included

20 Kt of TNT = $.84X10^{21}$ ergs
= $7.94X10^{10}$ BTU
= $20X10^{12}$ calories

Immediate Danger from Explosions

H. E. blast and throw-out

N. E. same, plus thermal and nuclear radiation

Thermal radiation is not emitted because the explosions are contained. The main immediate danger is from falling debris and, over a bigger distance, from shock transmitted through the air. Fifteen miles will eliminate any actual danger of these types, if 10 Mt is upper limit on yield.

V. Excavation Technology - Technical Questions

1. How does crater size depend on geological properties?
2. Can data on crater size, seismic effects, acoustic waves, and radioactivity distribution of low yield experiments be extended to megaton range?
3. How do nuclear charges in a row interact?

4. Can projects for nuclear excavation of channels through terrain varying in rock type and elevation be designed with confidence?

VI. Interoceanic Sea Level Canal

Atlantic-Pacific Interoceanic Canal Study Commission

Investigation by AEC
U.S. Army Corps of Engs.
Panama Canal Co.

Routes:

Route 17 - Sasardi - Norti - Darien - Panama

Route 25 - Atrato, Truando - Choco - Colombia

VII. Crater Design - Parabolic Approximation

PROJECT CARRYALL

J. Rios-Carvajal

SOURCES OF INFORMATION

1. Fry, John G.; Stone, Ray A.; and Crutchfield, William H., "Preliminary Design Studies in a Nuclear Excavation Project Carryall." Presented at the 43rd Annual Meeting of the Highway Research Board, Washington, D. C., January 13-17, 1964. Abridgment in UCRL-7632, pp. 11-16.

2. Kruger, Paul, "Nuclear Civil Engineering," Technical Report No. 70, Department of Civil Engineering, Stanford University, Stanford, California, pp. 261-263.

3. Zodtner, Harlan, "Operating and Safety Problems Associated with a Nuclear Excavation Project," Presented at the 43rd Annual Meeting of the Highway Research Board, Washington, D. C., January 13-17, 1964. Abridgment in UCRL-7632, pp. 17-20.

4. Prentice, H. C. and Peterson, E. T. L., Jr., "Construction and Feasibility Associated with Nuclear Excavations." Presented at the 43rd Annual Meeting of the Highway Research Board, Washington, D. C., January 13-17, 1964. UCRL-7632.

5. Talley, W. K., Railroad Engineering.

I. <u>Rerouting</u>

a. Expansion of highways and railroads have been limited by tools and equipment available.

b. Changes in routes pay through savings in fuel, wages, reduced time, reduced distances, etc.

II. <u>Project Carryall</u>

a. On a portion of the transcontinental main line, 165 miles long, between Needles and Bastow, California, there is a 78 miles stretch of railroad that was to be relocated. (Between the stations of Goffs and Ash Hill, where the line has a deviation southward to eliminate passage through the Bristol Mountains - a rather high, narrow chain of mountains in the middle of the Mojave Desert.

b. Present line deficiencies

1. train retarded by 3-deg curves

2. ascending grades of 1.46 percent

3. 2000 ft of elevation lost in dropping from an elevation of 2600 ft at Goffs to a low point of 600 ft at Cadiz and then rising to an elevation of 1900 ft at Ash Hill.

c. Cutting through the mountains on a direct line would:

1. shorten the distance 15 miles

2. maximum grade would only be 1 percent with a maximum curvature of only 1 deg.

3. would save an hour in travel time

d. Passage would require a 2 mile long tunnel (using conventional means for its construction).

1. Main disadvantage: expensive and costly to maintain

e. California Division of Highways

1. was investigating possible shorter routes for U. S. Highway 66, which will become Interstate Highway 40.

f. Joint feasibility study for the purpose of utilizing nuclear excavation technology was undertaken by the railroad company and the highway division with the technical assistance of the AEC and the Plowshare Division of UCRL. (Code name - Project Carryall.)

III. Details of Project Carryall

a. The cut was to be about 2 miles long with a maximum depth of about 360 ft and a top width ranging from about 600 to 1300 ft.

b. A total roadway width of about 330 ft was to be provided with the railroad (double track) located along the toe of the southerly slope, the eastbound highway (2 lanes) through the center, and the westbound (2 lanes) at the toe of the northerly slope. Ultimate expansion of the highway to a total of eight lanes was taken into consideration.

c. Geology of the area

   1. consists of soft volcanic rock underlain by meta-granite bedrock.

d. Nuclear excavation design contemplated the use of 22 nuclear devices ranging in yield from 20 to 200 kt (total yield: 1730 kt), and arranged in a row (would probably have been fired in two detonations.

e. A drainage problem was to be solved by trapping the flow in a separate crater, made upstream from the channel with a 100 kt nuclear explosion.

f. Radiological Safety

   1. The cloud of dust resulting was estimated to possess low radioactivity levels, not sufficient to be hazardous.

   2. Entry for an 8-hour work day should have been possible within 4 days.

g. Close-In Air Blast

   1. Town of Amboy - major problem but still no damage was expected

h. Long-range air blast

   1. below the threshold of damage

i. Ground shock

   1. Some minor damage, such as cracked plaster was expected at Amboy.

   2. Another problem - 900-psi gas line located 2 1/2 miles south of the cut.

IV. Construction Costs (from Nuclear Civil Engineering by Paul Kruger, Technical Report No. 70, Dept. of Civil Engineering, Stanford, California)

a. Conventional methods:

RR tunnel (12,800 ft)	4.34 mi	$14,552,000
Freeways	18.03 mi	7,170,000
		$21,722,000

b. Nuclear Explosives:

Preliminary investigations	330,000
Pre-shot construction (holes)	2,289,000
Nuclear excavation costs	1,940,000
Post-shot construction of RR	2,874,000
Post-shot construction of Highway	6,332,000
	$16,765,000
23 devices @ $0.5 M (est'd)	11,500,000
	$28,265,000

V. Cancellation

a. Project Carryall was cancelled.

CHARIOT PROJECT

N. Beylerian

HARBORS:

In order for nuclear devices to be economical as civil engineering tools, a project must be of extremely great magnitude. A large scale canal, a mountain pass, a harbor are at this time the most likely targets of application of nuclear energy. We shall concentrate on a harbor project - PROJECT CHARIOT.

Unfortunately, natural harbors do not occur where they are needed most, although man has adjusted himself to this fact by settling where they do occur naturally. Nevertheless, new horizons open, and regions hitherto hostile have to be exploited for their wealth. Since sea transportation remains the least expensive means of transportation, harbor facilities are needed early in the development of an area.

The choice of a site for a future harbor, and its method of construction will be decided after weighing several factors, such as cost, safety, urgency, and a host of intangibles (such as politics). From an engineering view-point consideration must be given to:

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a. expanding already existing facilities, if any,

b. improving existing natural features, such as deepening and widening of a river mouth,

c. lowering the ocean bottom,

d. construction of wave breakers,

e. carving a harbor from the land,

f. maintenance problems,

g. other considerations.

As for the choice of the method of construction - that is, using nuclear devices or not - we may note that under only few conditions are nuclear devices considered feasible at this time. However, some times those conditions do exist, and nuclear energy may yet prove to be the practical solution for harbor construction on the West Coasts of Alaska, South America, or Australia. Some of the reasons common to those areas that make this method of construction practicable are:

a. Freedom of choice of site, rather than dependence on natural features,

b. Sparse population; making relocation costs minimal,

c. Generally deep ocean approaches (or freedom from excessive sedimentation). We may note that cratering under the ocean floor is not an economical process, since craters will be shallow and wide. It was considered on the Eastern Cost of Australia, and abandoned.

d. A need for a harbor.

The Alaska site satisfied the first three considerations in general, but its use and need were questionable. It would probably become an important area for the population in time, and fishing boats would find refuge in the harbor in time of need. But those were not considered to be enough, and no clear evidence was seen that a harbor would in any appreciable measure boost the economy of the area. The area has no mining prospects or timber lands.

LOCATION IN ALASKA AND GEOPHYSICAL CHARACTERISTICS OF THE AREA:

The particular site chosen on the Western Coast of Alaska has appealed to engineers for the following reasons:

a. It is centrally located, that is to say, it would serve a coastline to the north and to the south of the site, from the Bering Straits to Barrow.

b. This site is relatively ice free, and the Chukchi Sea is relatively calm in this area.

c. The area is thinly populated.

d. The approaches are naturally deep enough for fishing boats sailing in the Chukchi Sea.

The geophysical characteristics of the area can be summed up as follows:

Ocean Characteristics: The Chukchi Sea is, in general, a rather shallow sea. It has a depth of about 50' at about 2 miles from the coast, and depths of more that 180' anywhere between Alaska and the Northeastern coast of Siberia are rare. Near the Chariot site, ocean depth reaches 15-20' immediately, then gradually reaches 50' at a distance of 2-4 miles.

Currents: Currents are northwesterly, about .5 miles/hr, paralleling the bottom contours. In general, surface currents seem to have the direction of the winds, when the latter are strong.

Coastal Features: Though the coast north of Point Hope is rapidly being eroded by strong winds and waves, the more protected shore near the Chariot Site doesn't seem to be subject to any important changes. However, during strong storms, depending on the angle with which the waves strike, beaches may be eroded rapidly affecting depths of up to 30' (hundreds of cubic yards per hour), to be deposited back when calm returns.

Floating Ice is found 8 months a year, nevertheless, the area is noted for its relatively large amounts of open water. In Juanuary and February ice may form very rapidly. Average thickness of ice may be taken to be about 3-5'. Exceptional cases occur when icebergs are brought to the Alaskan coast by high winds.

Climate: This region of Alaska is in the Arctic Tundra belt that covers the northern coasts of Alaska, Canada, and Siberia. It has a rather severe climate with eight months of winter and four months of spring type weather. In winter, low temperatures may be -40°F and high temperatures around freezing. In summer, freezing temperatures may occur any day, high temperatures over 70°F are also possible. In winter, there are 26 days without a sunrise, and in summer there are 54 days without a sunset.

Winds: In winter, northerly winds may reach gust velocities (over 80 mph), even in summer 30 mph winds are not uncommon. Storms may be expected half the time even in summer. Surface winds seem to have a depth greater than 1500', of great importance to us, since any fallout will depend on it.

Hydrology: Average annual precipitation is about 8" in the form of snow, that will melt in summer to form the many rivers and creeks of Alaska. In the Chariot site the Ogotoruk Creek may discharge as much as 1260 cu ft/sec at its height. The summer average is 60 cu ft/sec. From October to May there is no flow.

Although not in any large quantity, groundwater is found to be present in the permafrost and under it; some wells and springs are supplied by those waters. The creeks, rivers, many lakes or ponds form part of the overall water situation. In summer, some rivers and wells are utilized to supply water for human consumption, but in winter snow is the primary source, as well as during hunting and fishing trips.

The Soil is mudstone (and sandstone) around the Chariot site. It is permanently frozen starting at a depth of 1' or 2' to a depth of about 1100'. Within 100 yards of the coast this depth is nearer 950'. This means that all devices would have to be detonated in frozen soil.

Tests near the surface show that frozen mudstone has about 12.5% moisture. It is thought to be somewhat less at burial depths.

The specific weight of this soil ranges from 2.5 to 2.74.

More exact data on porosity, water content, etc., should be available by now.

BIOENVIRONMENTAL FEATURES:

It is found that the Chariot site was almost continuously habitated by the Eskimos from the middle of the 18th century until recently. Nowadays, the nearest communities are at Point Hope (32 miles, pop. 300), Kivalina (40 miles, pop. 140), and Noatak (75 miles, pop. 270). The Eskimos have traditionally been hunters; they still depend on hunting for their food. Fishing in the Chukchi Sea seems to be a recent development. The many lakes and ponds are not reliable in providing fish in sufficient quantities.

However, this area has surprised researchers by the number of species of all types of living creatures that exist here. Out of 60 known species of mammals in Alaska, 31 were found in the area (polar bear, walrus, seals, whales, caribou). Fifty-five species of freshwater and marine fishes, 300 species of flowering plants, 120 species of inland birds, and more than 1400 species in other groups also exist here.

Land animals are few in number (hundreds of caribou), but nine species of birds have a population of about 1/4 million. These birds nest in the sea cliffs within 8 miles of the Chariot site, the nearest one being within two miles of ground zero.

At this time it must be acknowledged that under the guidance of the AEC more than 40 separate investigations were conducted to obtain information on such varied features as:

Climatic cycles in the atmosphere and in the soil,

Geological and hydrological features,

Mechanical properties of the soil,

Chemical composition of the soil and bodies of water (contents of calcium, magnesium, potassium, sodium, strontium, etc.),

Complete ecological surveys of the land, the bodies of water, and the air, Radiological analyses of all types of living organisms, and terrestrial materials. (Such as determination of radioactivity and strontium-90 and cesium-137 levels in Alaskan animals, soil, and air.

DESIGN CHARACTERISTICS:

The harbor would be carved out of the land by detonating four 20 Kt devices to make an entrance, and one 200 Kt device inland was expected to provide a circular harbor of 1800' diameter. However, recent tests have indicated that the harbor would be closer to 1500'.

The first 20 Kt device would be placed near the mouth of the Ogotoruk Creek, at about 200' from the coast, at a depth of 400'. The other three 20 Kt devices would be placed in a northerly direction at distances of 500'. The 200 Kt device was to be placed at a distance of 900' from the last 20 Kt device in the NW direction, at a depth of 800'.

a. Taking the Depth of Burial as the basis, and using Figs. 4-50 and 4-51 (CUNE) with $400/20^{1/3.4} = 50.6$ m/$kT^{1/3.4}$, obtain

Apparent Crater Radius: 48x2.41x3.28 = 380'

Apparent Crater Depth: 25x2.41x3.28 = 200'

Other Data: Lip Height: 50'

Depth of True Crater: 520'

Radius of True Crater: 450'

For the 200 kT device all those values are doubled.

Spacing of 500' between the 20 kT devices will ensure a continuous ditch.

b. No thermal effects are expected.

c. Air Blast: Fig. 4-55 (CUNE) may be used to draw the maximum air blast vs distance curve for the total 280 kT charge. In Fig. 4-55 (CUNE), the abscissa is divided by $280^{1/3}$ and the ordinate is divided by 5, since air blast seems to be only 20% of what it would be if the blast were in the atmosphere. See accompanying graph.

d. Seismic Disturbances: In alluvium the surface velocity is given by Equation 4.3-2 as

$$v = .015 \frac{Y^{2/3}}{R^{3/2}} \text{ cm/sec} \quad \text{for} \quad 18 \text{ km} \quad R \quad 350 \text{ km}$$

at closer ranges (from the Gnome shot)

$$v = .422 \left(\frac{R^{-1.65}}{Y^{1/3}}\right) \qquad \text{in salt beds. Eqn. 4.3-6}$$

Thus, for the arbitrary velocity of 10 cm/sec

R = 6 miles using Eqn. 4.3-6

R = 10 miles using Eqn. 4.3-2 (not applicable)

If the 10 cm/sec velocity is accepted as the plaster cracking limit, then no damage would be expected in any of the settlements in the neighborhood, especially since no plaster exists there.

e. Throwout (very speculative)

Downwind 1/4 mile almost 120"

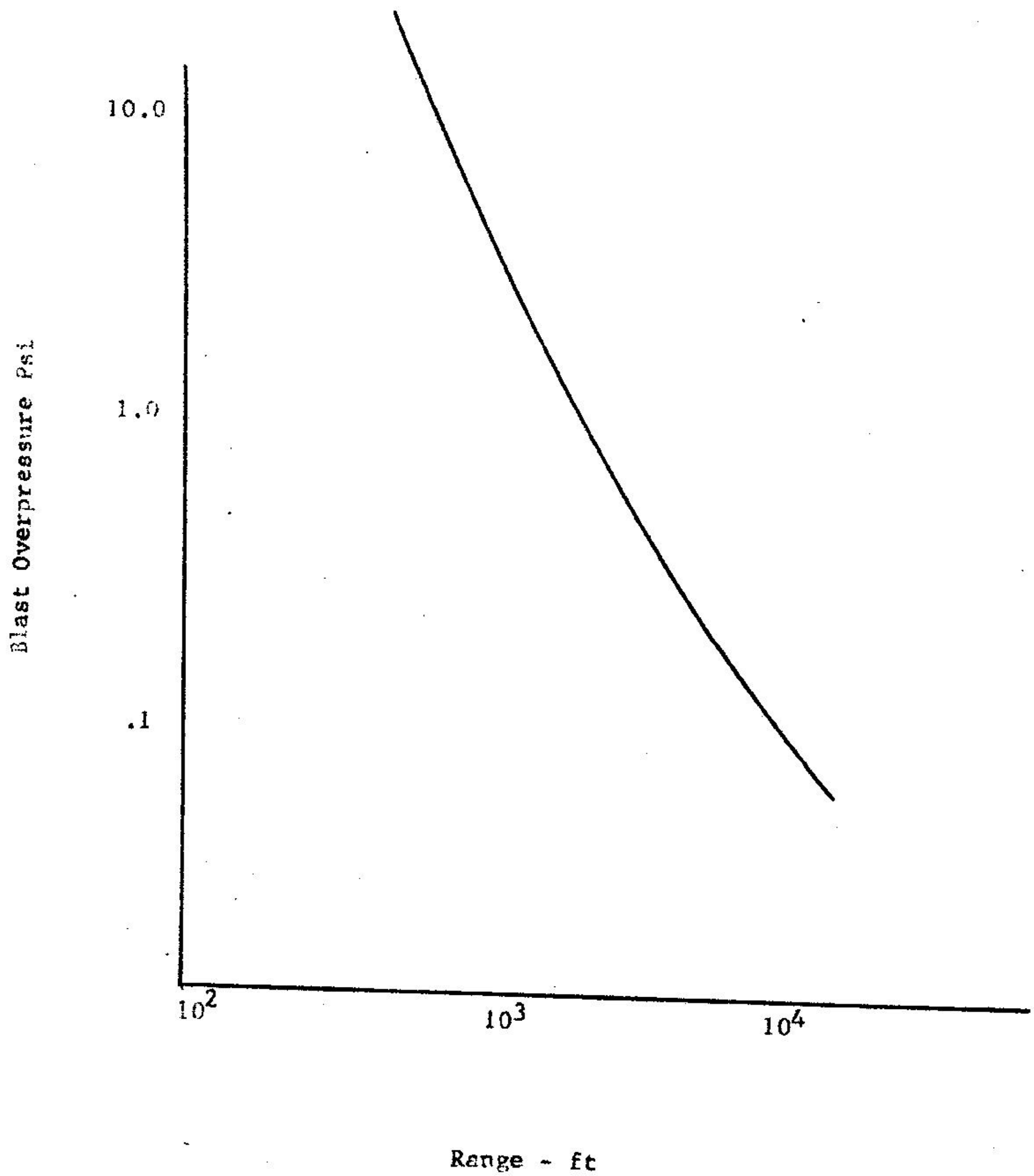
Crosswind 1/4 mile 40"

Downwind 3/4 m. almost 1/4"

Crosswind 3/4 m. .04"

f. Fallout: Approximately .5% of total produced radioactivity would be carried by winds (8 knots), and 80% of this would fall within 20 miles downwind.

The explosions were initially scheduled so that fallout would be over the land. However, those winds were not considered reliable, and finally it was agreed upon that northerly winds would be utilized despite inherent difficulties in collecting samples after the event.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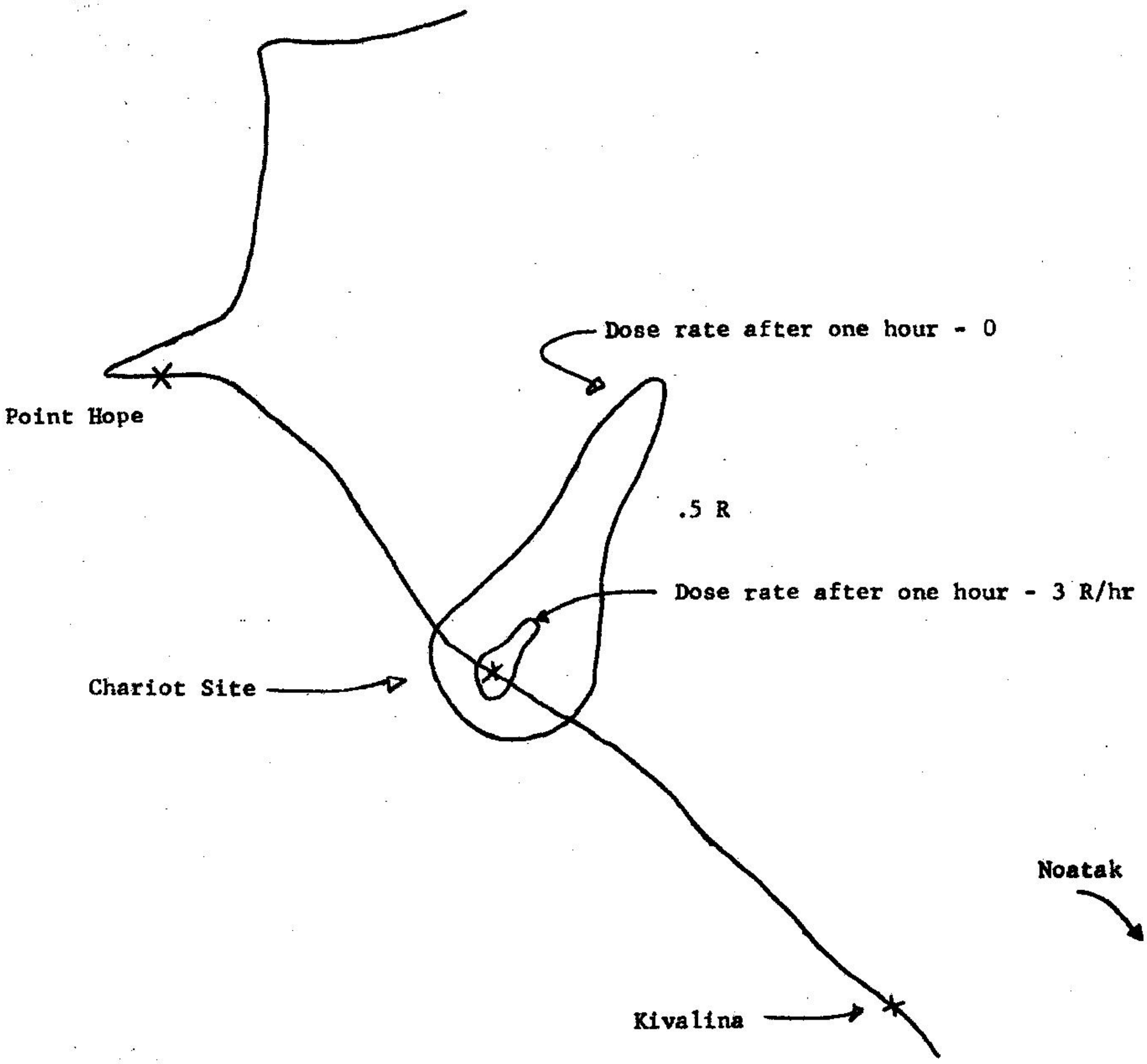

Estimated Dose Contours

PROJECT CHARIOT:

If carried out as planned, the following would be a summary of effects, as anticipated:

a. There would be no need to evacuate any of the settlements,

b. No structural damage was expected due to seismic effects or air blast. Though based on statistical analysis, some windows might be broken as far away as Kotzebue,

c. In the immediate vicinity of the site, all life might be assumed destroyed within a 1/2 mile radius. In case winds were seaward, destruction of marine life would be smaller, owing to the cushioning effect of the sea, and already sparse marine life. Plants would be buried under throwout material in this area.

d. The greatest ecological change might be due to slides of the sea cliffs harboring the five colonies of birds. Particularly, if the tests were performed during the nesting season, damage to eggs might effect important changes in the bird population.

e. Due to lack of experiments under similar circumstances, conditions at the entrance to the harbor can't be estimated yet. In extreme cases, the entrance may be blocked completely by throwout material, in which case it could take several years for water to seep into the craters. On the other hand, the entrance might also be completely open. In that case, it would take the ocean a few hours to fill the harbor.

f. The Ogotoruk Creek would be dammed about 2000' upstream, thus forming a lake. Another dam would be created at the present location of the Ogotoruk Creek mouth. Either a second lake would form, or the creek would erode some of the present coastal zone and form a lagoon, as is the case presently.

CONCLUSION:

The Chariot Project was shelved several years ago. Thus: engineers have been denied the chance to apply nuclear energy to a useful project at a very small ecological cost; biologists were deprived of the opportunity to observe the changes induced by an instantaneous biological change, and also note the development of life in a region where new lakes and creeks would form, following the formation of the harbor.

PROJECT GASBUGGY

D. Taylor

Coal and fuel oil are used to produce 60% of the electricity in the U.S. (1). The burning of this fuel contributes about 44% of the SO_2 emitted to the atmosphere. In 1963 this amounted to over 10 million tons of SO_2 (2).

The Health, Education, and Welfare Department has already set tight sulfur content levels on fuels burned by federal installations in some major cities. It has been estimated that if the sulfur emission standards of fuel restrictions now recommended by HEW were to be made effective at this moment throughout the U.S., more than 90% of the 240 million tons of coal presently purchased by electric utilities could not be consumed (1).

There are four possible ways to solve the problem of SO_2 emission.

1. Nuclear Power. The use of nuclear power is expanding at a very rapid rate. About one half of the new power plants being ordered are nuclear. Some estimates forsee that nuclear energy will produce 20% of our electricity by 1980. Even then, increased demands for power will cause a 75% increase in fossil fuel consumption.
2. Sulfur dioxide removal from stack gases. Much research has been done on this aspect of the problem, but a large scale, economic solution has not been found.
3. Removal of sulfur from oil and coal before it is burned. For oil this is possible but expensive. The Caribbean refineries, which treat crudes with a high sulfur content, would be required to double their equipment investment in order to produce residual fuel oil with 0.5% sulfur content (1). The 20 to 60% of the sulfur that is organically bound in coal cannot be removed.

4. Replace coal and oil with natural gas. Natural gas is low in sulfur content. It is presently used to produce 22% of our power. But supplies are short. Our reserves in 1967 are at the lowest value in 21 years. We only have a reserve of 16.5 years. If New York City alone were to use natural gas instead of coal and oil, an increase in the total supply of gas of 260% would be required.

To anyone familiar with the problem of increasing natural gas production, nuclear stimulation of gas wells is a very attractive possibility. Some estimates say that nuclear stimulation would immediately double our reserves of natural gas. Companies feel that nuclear stimulation is a worthwhile investment. If pollution standards were set and enforced, the value of natural gas would probably increase and make nuclear stimulation even more valuable.

In an idealized, cylindrical model of a gas reservoir the gas flows radially from the reservoir into the well bore. A given particle of gas must flow through every cylindrical element of the reservoir. At the outer boundary of the reservoir, the resistance to flow is small because the area available for flow is quite large. But near the well bore the resistance is considerably larger because the area is drastically reduced. The purpose of nuclear stimulation is to remove as much of the resistance as possible by enlarging the radius of the well bore.

Let us show why this is true. One begins with Darcy's Law for flow through a porous medium.

$$v = \frac{K}{\mu} \frac{(-\Delta P)}{(\Delta x)} \qquad (1)$$

v = velocity of flow based on the entire area

K = permeability of the medium

μ = viscosity of the fluid

ΔP = the pressure difference across the element, Δx

Δx = the thickness of the medium

As $\Delta x \longrightarrow 0$,

$$v = -\frac{K}{\mu}\frac{dP}{dx} = -\frac{K}{\mu}\nabla P \qquad (2)$$

For cylindrical coordinates, radial flow only,

$$v = -\frac{K}{\mu}\frac{dP}{dr} \qquad (3)$$

In the derivation of the equation describing the flow in the reservoir, the following assumptions will be made.

1- Cylindrical model

2- Laminar flow

3- Constant viscosity

4- Uniform temperature throughout the reservoir

5- Constant permeability

6- The equation of state of the gas is

$$PV = ZRT \text{ or } \rho = \frac{PM}{ZRT} \qquad (4)$$

If we examine a period of time that is relatively short compared to the total production time, we can consider the reservoir as being at steady state. Then the following continuity equation must hold.

$$m = \rho Av = \text{Constant} = C_1 \qquad (5)$$

Where

m = mass flow rate of natural gas

ρ = density of the gas

Substituting into the continuity equation the proper expressions for ρ, A, and v gives

$$\left(\frac{PM}{ZRT}\right)\left(2\pi rh\right)\left(-\frac{K}{\mu}\frac{dP}{dr}\right) = C_1 \qquad (6)$$

h = height of the reservoir

T = temperature of the reservoir

Separation of variables and integration gives

$$m = C, = - \frac{M \pi h\ K(Pw^2 - Pe^2)}{ZRT\ \mu\ .\ \ln\ (r_w/r_e)} \tag{7}$$

Where

r_w = radius of the well bore

r_e = effective radius of the reservoir

Pw = the pressure at the well bore

Pc = the pressure of the reservoir at r_e.

Conversion of the mass flow rate to a volumetric flow rate at standard conditions and subsequent combination of all constant terms gives

$$Q = C \frac{hK\ (Pw^2 - Pe^2)}{\mu T\ \ \ln\ (r_w/r_e)} \tag{8}$$

From Equation (8) it is evident that the production rate can be increased either by decreasing the pressure in the well bore or by increasing the radius of the well bore. A decrease in P_w increases the pumping cost and becomes unreasonably expensive before substantial gains in production rates can be obtained.

The size of the bore can be increased in the following ways:

1. Drill a larger hole. This is very expensive.
2. Conventional explosives can be used to fracture the rock around the well and increase the effective radius. Increased production rates of five times that obtained without shooting have been achieved.
3. Hydraulic fracturing. A liquid under high pressure is pumped into the reservoir to fracture the rock. Sand is mixed with the liquid to hold the cracks open after the pressure is released. Hydraulic

fracturing sometimes increases initial production rates by a factor of 10. In low permeability reservoirs production gains are rapidly lost after an initial flush in production caused either by conventional shooting or by hydraulic fracturing.

4. In limestone reservoirs the size of the well bore can be increased by pumping HCl into the hole to dissolve the rock. This is clearly limited in possible applications and is very expensive.

5. Nuclear Stimulation

The U. S. Atomic Energy Commission, the U. S. Bureau of Mines, El Paso Natural Gas Company, and the Lawrence Radiation Laboratory have investigated the possibility of stimulating a gas well with nuclear explosives. In October of this year the first actual test of the program, Gasbuggy, will be conducted in northern New Mexico. (Data for Project Gasbuggy are given in Reference (3).) The site selected is in the central section of the San Juan Basin. The gas bearing portion is called the Pictured Cliffs formation. This formation is 300 feet thick between depths of 3850'. The site is remote, but accessible. The nearest house is five miles away and the nearest city, Farmington, is 70 miles away. The population of Farmington is 23,000. There is enough drilling in the area so that the reservoir is well classified as to gas content, permeability, etc., but only one gas well is near enough for shock damage.

Another advantage of the location is the fact that the people and government of New Mexico have accepted nuclear testing. The conservation commission will probably permit some shifting of allowables between other producing wells in order that maximum production can be studied.

There will be two preshot holes, 100 and 200 feet from the emplacement hole. These are to confirm.

1. There is no mobile water in the vicinity of the shot.

2. The gas content and permeability at the site

3. The performance of an unstimulated well

These two holes will also be used to study the extent of fracturing.

The shot will be at the bottom of the Pictured Cliffs formation which consists of sandstone and shale. The proposed device is to have a yield of 10 kilotons. The expected effects of the detonation are given in the following table:

Cavity Radius	65'
Rubble Chimney	300'
Vertical Fracturing	390'
Fracturing Radius	195' (conservative)
	430' (optimistic)

The predicted gas recovery over a total production period of 20 years and with 160 acre well spacing is expected to be

Conventional Stimulation	537×10^6 ft^3	10% of total gas present
Nuclear Stimulation (conservative)	3520×16^6 ft^3	67% " " "
Nuclear Stimulation (optimistic)	3748×10^6 ft^3	71% " " "

A study (4) was made of the economics of nuclear stimulation. The study was made for the proposed Dragon Trail project to be conducted by Continental Oil Company. In this particular case, it is projected that a well stimulated by nuclear explosives will produce about the same amount as four conventional wells. These four conventional wells cost $100,000 and the return on the investment is 29%. The return for a nuclear stimulated well depends entirely on the cost of the device. If the figures given in Reference (4) are used with an estimated cost of $50,000 for drilling the emplacement and the re-entry holes, the following relationship between the cost of the device and the return on the total investment is obtained.

Cost of device	Return
$600,000	3%
$225,000	16%
$115,000	28%

The AEC has announced a price of $400,000 for the 40 kiloton device to be used in this application (5). This would give a return of somewhat less than the 29% for conventional wells.

The attractiveness of nuclear stimulation would be considerably greater if the cost of the device could be lowered. There are several sedimentary basins in the Rocky Mountain region that have more gas than Pictured Cliffs. Conventional stimulation cannot recover it. If Gasbuggy proves successful, this region alone could use 30,000 devices. If they were produced in such great numbers, the cost of nuclear explosives would surely decrease.

POSSIBLE DANGERS:

Contamination of the atmosphere by venting radioactive gases through fractures or through a failure of the stemming in the emplacement hole is considered remote based on the experience of previous tests. Shoal had a yield of 12.5 kilotons of a depth of 939 feet. There was no venting. We can compare these figures with those for Gasbuggy whose yield and depth are to be 10 kilotons and 4150 feet, respectively.

All geologic studies of the area indicate that there is no mobile water. These conclusions will be checked by the two preshot wells. The nearest water wells are 50 miles away and take their water from a level 1700 feet above the test site for Gasbuggy.

If there is an aquifer near the formation of interest and either the rubble chimney or extensive fracturing breaks through to the water, there is a possibility that the cavity and rubble chimney would be filled with water thus rendering the well useless. No trouble of this type is expected in

Gasbuggy.

For the 10 kiloton device to be employed in Gasbuggy, residential plaster would crack within 1.6 miles. There is only one gas well near enough (600 feet) to be damaged by the blast. This well will be considered a part of the experiment to determine the range and extent of the shock damage.

Radioactive solids will not be a problem. Nearly all of them will be trapped in the melt at the bottom of the cavity. The gases that will be troublesome in an all-fission explosion are

	1/2 life
Krypton 85	10.3 years
Xenon 133	5.3 days

Iodine 131 will be present, but it can be removed by decontamination procedures. After about 10 months the radioactivity from Xe and I would be negligible, but Kr^{85} would be 690 times the amount permissible to the general public.

The first step will be to bleed the well and dispose of the initial radioactive gases. This might be done by burning. The best dilution in the rubble would be accomplished by evacuating the well bore and letting it refill in cycles. In this way the entire volume of the chimney would be used to dilute the radioactive gases.

Another dilution will occur when the produced gas is fed to pipelines carrying gas from other sources. The dilution here is about 1000 to one. As soon as the mixture (gas from Gasbuggy plus gas already flowing in the line) has a radioactivity low enough for LEC (license Exempt Contamination), Gasbuggy can begin producing. The level of radioactivity permitted to the general public is 1/30 of the LEC. But when natural gas burns in a house under the worst possible conditions it is diluted by a factor of 200 to 1. The resultant radioactivity would be 1/7 of that allowed the public.

FUTURE TESTS:

If Gasbuggy is successful, it is presently planned to conduct Dragon Trail about three months later. This test will be conducted by Continental Oil Co. The test site is in a formation 400 feet thick at a depth of 2700 feet in western Colorado. After Dragon Trail will come Project Rulison (6). It will be in the very deep, thick, impermeable Mesaverde formation in Colorado. The plans for Rulison specify two vertically placed shots separated by 1000 feet in elevation.

Possible improvements of nuclear stimulation beyond these three tests are:

1. The development of explosives having a diameter of seven inches. This would permit the use of existing wells as emplacement holes.
2. The development of a technique for directing the force of a shot into the most productive formations. This might be done by an array of devices fired simultaneously or in sequence.

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MINING APPLICATIONS

F. Muñoz-Ribadeneira

I. Introduction

The possible applications of Plowshare technology for copper mining that have appeared in the literature are:

1. Preparation for open-pit-mining: removal of overburden
2. Preparation for block-caving, shattering the ore body to facilitate removal
3. Preparation for in-situ leaching, rendering the ore body permeable to leaching

Block caving and leaching in-situ are to be discussed in this section. We are going to concentrate on leaching in-situ due to the fact that it eliminates major disruption of the geological systems at the surface, since no overburden need to be removed nor solid gangue dumped. Spent leaching and possible cementation solutions must be disposed of, but, if the leached liquor can be made rich enough and the ore body is large enough to make electrowinning practical, waste products can be reduced to a minimum.

II. Requirements For a Successful Leaching Operation

As an example, if the ore body has copper:

1. An ore body of sufficient size
2. Copper content of at least 0.3%
3. Ore body composition and structure which allows physical penetration of the leach solution
4. Low acid consuming gangue
5. A contained environment which provides for recovery of the leaching solutions.
6. Copper present in a readily soluble physical and chemical state

These conditions either must exist initially or should be achievable by reasonable engineering procedures. Further the following supplies and services must be available.

7. Sulfuric acid or other suitable leach liquors at low cost and in adequate supply.
8. For concentration, thin iron sheets
9. Liquor which is rich enough and abundant enough to justify the capital investment in electrowinning of the copper.

Now, if we consider in this process the application of Plowshare in preparation for in-situ leaching, the following requirements should be fulfilled.

10. An ore body that extends to sufficient depth to permit the deep emplacement of the explosive.
11. Substantial overburden to assist entrapping any radioactive effluent and make the ore body relatively unattractive economically for any other technology.
12. An ore body of significant size to allow for significant tests, such that the loss as a result of the test would be significant in the total copper resources.
13. Location where the combination of deep emplacement, heavy overburden, low surface population density with few surface structures would make the potential damage by the test shot reasonably small.

We will see later how we can meet all these requirements.

III. Copper Bearing Area in Puerto Rico

The known copper deposits in Puerto Rico are found along the south western edge of the Utuado pluton which is an intrusive complex of granodiorite, quartz, diorite, and minor gabros.

The copper mineral is chiefly chalcopyrite ($CuFeS_2$) and there are also chalcocite (C_2S) and Covellite (CuS). To the northward the cretaceous-Eocene wells including the Pluton are overlain by a thick series of limestone sediments. The copper ore is located in a zone of hydrothermal alteration. The general structure trend is WNW-ESE.

A test hole for oil was drilled to 6434 ft at a point of about 10 Km east of Arecibo, near the coast and this penetrated 5580 ft of limestone sediments before encountering volcanic sandstones or pluton rocks formation. This is the only direct evidence that the older rocks continue under the cover of the limestone sediments. This does not necessarily imply that the pluton and the accompanying hydrothermal zone continues any considerable distance northwestward. There are also the probabilities that it may end in a short distance or that it may increase in size.

In Puerto Rico we have to consider the following facts:

1. Limestone does not appear to be mingled with the ore body.
2. The structure tends fairly deeply as far as 5000 ft and the overburden might be turned to advantage.
3. Chalcopyrite is a very insoluble copper ore in diluted sulfuric acid solutions, but shock-heat effects may change completely the solubility problem. This is uncertain as well as is the physical location of a suitable ore body and characteristics.
4. Land evacuation, shock and earthquake effects, as well as socio-political problems may exist.
5. Pollution problems are greatly reduced, and no damage to landscape is expected, but both problems are becoming a great issue in Island politics.

In the case of leaching in-situ and in block caving operations the nuclear explosions are completely contained underground.

IV. Considerations of Nuclear Effects

As a result of a completely contained nuclear explosions a short-lived cavity would exist. The roof of the cavity would collapse and the caving would progress vertically upward. We are interested in knowing the dimensions of the cavity and chimney, the properties of the material inside the chimney, the economics regarding leaching in-situ of the copper, seismic effects, and possible ground water contamination.

A. Dimensions of the cavity, chimney and other parameters

The dimensions of the cavity are given by the following formula:

$$R = C \frac{W^{1/3}}{(\rho h)^{1/3\gamma}} \qquad (1)$$

where

γ = 4/3

W = yield

ρ = density

h = overburden

That reduces to

$$R = C \frac{W^{1/3}}{(\rho h)^{1/4}} \qquad (1\text{-a})$$

C is a constant depending on gamma, or on the quality of environment in which the device is exploded. The height of the chimney is related to the radius of the cavity by the formula:

$$H = K R \qquad (2)$$

Where K is another empirical constant.

The depth of burial for which venting is prohibited is given by:

$$Dob = 450\ W^{1/3}\ (ft) \qquad (3)$$

Calculations for depth of burial with respect to yield of explosives are given below:

TABLE 1. Calculations of Depth of Burial

W	1	5	10	25	100	200
$W^{1/3}$	1	1.71	2.16	2.93	4.65	5.85
D, ft	450	770	973	1320	2160	2630

It is clearly seen that we need a minimum depth of about 1000 ft for a 10 Kt explosion. Since we are interested in using the highest possible yield without causing seismic damage we will use 950 mts as Dob and calculate the cavity and chimney dimensions using the following parameters.

C = 59.0

K = 4.35

ρ = 2.70

Dob = 950 m (3000 ft)

TABLE 2. Calculations of Cavity Chimney Dimensions and Economics

(Kt) W	1	5	10	25	100	200 Kt
$W^{1/3}$	1	1.71	2.16	2.93	4.65	5.85
m R	9.1	15.50	19.6	26.8	42.4	53.2
m H	39.6	57.50	85.5	116.	184	232
$\pi r^2 h$ m^3 V	$1.0x10^4$	$4.3x10^4$	$1.04x10^5$	$2.16x10^5$	$9.25x10^5$	$1.87x10^6$
Ton	$2.7x10^4$	$1.18x10^5$	$2.76x10^5$	$5.85x10^5$	$2.64x10^6$	$5.65x10^6$
Kg	$2.7x10^7$	$1.18x10^8$	$2.76x10^8$	$5.85x10^8$	$2.64x10^9$	$5.65x19^9$
1% Cu	$2.7x10^5$	$1.18x10^6$	$2.76x10^6$	$5.85x10^6$	$2.64x10^7$	$5.05x10^7$
Recup.50% Cu	$1.35x10^5$	$0.59x10^6$	$1.38x10^6$	$2.95x10^6$	$1.32x10^7$	$2.05x10^7$
Price $.924/Kg	$1.25x10^5$	$0.54x10^6$	$1.27x10^6$	$2.72x10^6$	$1.22x10^7$	$2.36x10^7$
Price N.E.	$3.5x10^5$	$3.50x10^7$	$3.50x10^5$	$3.9x10^5$	$4.5x10^5$	$4.8x10^5$
Profit*	No	No	Yes	Yes	Yes	Yes

*For economic profit we consider a ratio of 1 to 3 between the price of the nuclear explosive to the total price of the copper recovered to be needed.

These calculations indicate that we need a 10 Kt explosion. This will require an ore body of at least 40 m wide by 90 m high.

B. Seismic Considerations

A damage threshold for plaster cracking of 8-10 cm/sec surface velocity has been agreed upon.

Due to the dense population of the island (about 300 persons/Km^2), five or ten Km is the maximum distance to which we may have the maximum surface velocity greater than 8 cm/sec. Let us consider the yields of explosives related to the surface velocities, calculated according to the following formula:

$$V = K_v W^{0.67} R^{-1.5} \qquad (4)$$

where

K_v = constant

V = Surface velocity

R = radius (Km)

W = yield (tons)

K_v is a function of the material = 0.082 for granite

TABLE 3. Calculation of Surface Velocities

W(ton)	$10x10^3$	$25x10^3$	$100x10^3$	$200x10^3$
$W^{0.67}$	470	1000	2250	3631
V_{10}	1.19	2.54	6.16	9.95
V_5	3.42	7.09	17.6	28.4

Displaced population in 10 Km = $\pi \times 10^2 \times 300$ = 94.200

5 Km = $\pi \times 5^2 \times 300$ = 23.600

These results basically indicate that, due to the island density of population, we are forced to use explosives with a maximum yield of 100 Kt for no damage located outside a radius of 10 Km, or a maximum yield of 25 Kt for a radius of 5 Km.

C. Characteristics of the Rubble Inside the Chimney

The chimney rock fragment size is a function of the cavity size, distribution of original faults and fractures, mechanical and thermal stresses applied by the explosion and the breaking and grinding action during collapses. The formation of rubble filled processes involves a number of random processes, so statistical techniques are appropriate in estimating the rubble size distribution.

In practice rubble fragments have been found to vary in size from that of sand or dust to a maximum dimension of approximately 1/4 of the cavity radius. The particle size distribution varies through the chimney with smaller particles concentrated at the sides and bottom of the chimney where crushing and grinding may be expected to be most pronounced.

The following operational formulas have been used for determination of the parameters needed in the calculation of the specific surface and permeability; assuming a logarithmic distribution the formulas are:

$$\sigma^2_{\ln D_W} = \sum_i^n \frac{W_i}{W} (\ln D_{W_i} - \ln \overline{D_W})^2 \qquad (5)$$

$$\ln \overline{D_W} = \sum \frac{W_i}{W} \ln D_{W_i} \qquad (6)$$

$$\ln \overline{D_{\sigma_W}} = \ln \overline{D_W} \qquad (7)$$

$$\overline{Dvsw} = \exp \left[\ln D_{\sigma_W} - \frac{\sigma^2_{\ln D}}{2} \right] \qquad (8)$$

This means that with formula (6) we know (5) and then (7). The calculated values are substituted in (8) and the average diameter in relation to surface/ volume is known. We also know that:

$$A = \alpha_s D^2 \qquad (9)$$

$$V = \alpha_v D^3 \qquad (10)$$

So the surface area to volume ratio is given by

$$A/V = \frac{\alpha s\ D^2}{\alpha s\ D^3} = \frac{\alpha s/\alpha V}{\overline{Dvs}} = Svs \tag{11}$$

Svs has the dimensions of cm^{-1}. If we divide it by the density, we have the dimensions of cm^2/gr. $\alpha s/\alpha v = 6$ for spheres and 7.7 for sharp particles. This latter value will be used since sharp rubble particles have been observed in tests. Therefore,

$$Svs = 7.7\ \overline{Dvs} \tag{12}$$

The permeability of rubble to fluid flow is given by:

$$K = \frac{\phi^3}{5(1-\phi)S_v^{\,2}} \tag{13}$$

Where K is the permeability and ϕ is the porosity.

Due to a nuclear explosion the effective porosity of the rubble is greater than the original rock giving as a result an increased permeability permitting penetration of the sulfuric acid solutions more easily.

GEOTHERMAL HEAT AND SALTWATER CONVERSION

K. Pedersen

Review and critique of an article by George C. Kennedy, University of California, L. A.

I. Availability of Geothermal Heat

The mean heat flow from the interior of the earth in continental North America is approximately 1.2×10^{-6} cal/cm^2 sec.

Areas with 5-10 times the average heat flow are known. One such area extends from the Easter islands in the Pacific into the southern part of the United States. This is an area approximately 50-100 miles wide and several thousand miles long. Whereas in an average area the temperature gradient is $\sim$1°C/100 ft, based on the heat flow and the ave. K for rock, in the areas of high heat flow the gradient must be $\sim$10°C/100 ft. These areas of high heat flow are exclusive of those with hot spring activity or recent volcanic action.

The latter show themselves by hot springs or steam rising from the earth. It is postulated that the mechanism for these cases is that water of a meteoric origin and at depths on the order of 1000 ft cools the magmatic body and carries the heat upward, thus heating large volumes of rock. The heat is stored in these enormous quantities of rock which may have volumes of 10's of cubic miles. In some of these hot spring regions temperatures up to 500°C may be found at 10,000 ft or less.

The heat of interest in a plowshare project is that which does not appear at the surface and is therefore not readily available by conventional means.

II. Availability of Nuclear Explosives

Since we are talking about a proposal for the extraction of this heat in areas where it does not appear at the surface, it is necessary that the heat bearing rock at large depths be broken up to provide a larger heat transfer area and at the same time smaller distances for thermal conduction and also to provide access for the cooling fluid, be it fresh water or salt water, and finally to allow the steam to get out. At these depths and with these energy requirements nuclear explosives are the only energy sources that can be considered.

Kennedy has worked out an example where a 5 Mton device is detonated at a depth of 10,000 ft, producing an initial cavity with a diameter of 1,000 ft, and a volume of 5×10^8 ft^3. With an assumed postshot porosity of 12% he calculates a rubble chimney height of 8,000 ft, leaving a 2,000 ft "cap" of relatively undisturbed rock. The calculation of the rubble chimney is incorrect because he has taken the total porosity created by the shot and converted it into a cylindrical rubble cone. It is a fact, however, that a large portion of the porosity is due to crack formations emanating radially from the shot and, therefore, not contributing to the chimney.

Extrapolation of Fig. 6.3 in CUNE by Talley et. al. tells us that we may expect a chimney height of $\sim$ 1,200 ft and Table 4.1 in CUNE allows us to estimate a maximum chimney height of 2,000 ft. Even these estimates are probably optimistic because Project "Handcar" gave an actual rubble cone which was $\sim$ 30% shorter than would be predicted from the "Scaling Laws".

The effect of the miscalculation of the chimney height may be to our advantage because it means that it is not necessary to emplace the device at 10,000 ft. Thus, provided we can find heat bearing rock at shallower depths, considerable savings may be realized.

Kennedy has calculated the energy available at temperatures over 100°C to be $\sim 1.8 \times 10^{16}$ cal. Since the heat liberated by the explosive is $\sim 5 \times 10^{15}$ cal. the total heat available is approximately 5 times the energy of the explosive.

This will produce $\sim 10^{11}$ pounds of superheated steam, or enough to generate 50,000 KW of electricity over a period of 10 years.

III. Cost of Steam

A direct comparison is made with steam plants operating on similar steam to obtain a value of the energy available from the cone.

The Pacific Gas and Electric Co. is buying steam from wells at the Geysers in Sonoma County, California. This steam is low-pressure, low-quality and costs $1/900# which amounts to 2 1/2 mills/KW Hr generated. The total cost of the power is 5 1/2 mills/KW Hr at the consumer.

The cost of the steam is essentially due to the cost of emplacement and the cost of the device. These are estimated at $4 mill. and $1 mill. respectively. Considering the worth of the steam, Kennedy reasons the project will be economic by a factor of 2. No cost has been considered for post-shot drilling which may have to be repeated several times, as crud forms inside the chimney. It is possible, however, that it may cost only half as much to emplace the device, since the rubble chimney will be but a fourth as high as calculated by Kennedy. This saving may indeed make the project even more economically feasible than first conjectured.

Since we are mining earth heat and not heat introduced by the device we may remove it at any desired rate and would actually have more heat available if it were removed at a slow rate. Energy may continue to be extracted after the first cone is exhausted by detonating a second device slightly more than one crater radius away from the first one. If we still introduce the water through the first cone we would presumably have preheating of the water.

IV. Problems Which May be Encountered

a) Ground motion associated with large nuclear detonations. Particularly with respect to consideration of sequence of shots.

Deep circulating water which may enter rubble and flash into steam. It will be necessary to know beforehand whether this possibility exists.

b) Behaviour of rubble cone as function of time. At 10,000 ft the pressure is approximately 600 atmospheres and some of the softer rocks would lose the induced permeability long before the heat could be extracted.

c) Radioactivity in steam. For a 5 Mton or similar device the energy would be supplied mainly from fusion. Considerable tritium contamination of the steam would occur. The tritium gives off a 0.018 Mev β which would present no problems in a closed-cycle power plant. If the steam were to be used for drinking water a secondary heat exchanger ($\sim$90% eff.) would be the simplest solution.

SALTWATER DISTILLATION

If we continue to use Kennedy's rubble cone the pressure would vary from $\sim$1680 psi to $\sim$ 8400 psi from the top of the cone to the bottom. Thus the thermodynamic conditions for flashing the saltwater into steam are particularly favorable. This is enhanced by the fact that at the temperature of the rubble (ave. $\sim$ 660°F) the constant temperature line of a T-S diagram is asymptotic to the saturation line at approximately 3000 psi. These two conditions coincide with the critical point. The advantage of the critical point lies in the fact that minimum energy would be required to flash the water into steam. If the "free energy" G is used to measure the amount of work done in the change, and $G = H - TS$ we see that since $H_{fg} = 0$ at the critical point, and the change in entropy is small because the entropy decreases with increasing pressure of the steam, the minimum amount of energy is expended near the critical point. Above the critical point, however, the water does not flash into steam, and since most of Kennedy's calculated rubble cone provides for pressures above the critical pressure all of the steam should be produced at that depth below the surface where the pressure coincides with the critical pressure. This will probably mean that all of the crud (including the salt) will settle out at that level and render the rubble impermeable.

SIMULATION - SIMILITUDE

I. Subject of Similitude can be based on:

a) similar equations (analogies) (necessary to be able to write diff. eqn.)

b) dimensional analysis

Dimensional analysis is in turn dependent on our system of measurements.

All general equations, in order to be valid, must be dimensionally homogeneous.

We may write any unknown quantity X in terms of its basic dimensions. Likewise all of the variables on which the unknown quantity depends may be written in terms of basic dimensions, and it can, therefore, be shown that the unknown quantity X may be written in terms of the independent quantities raised to appropriate powers. Thus $X = f(I_1^a, I_2^b, I_3^c, \text{etc.})$, where I stands for independent quantity.

According to the Buckingham Pi theorem we may express a set of M quantities which have N dimensions as a functional relationship between M-N dimensionless terms called Pi terms. Thus $\pi_1 = f(\pi_2, \pi_3, \pi_4 -- \pi_{M-N})$ where usually the unknown quantity X will be contained in π_1.

II. Difference Between Modeling and Scaling

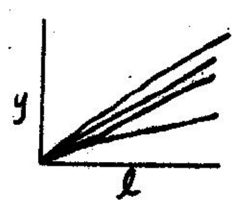
By scaling is usually meant that a certain constant relationship exists between all of the variables in an expression. However, as we have seen with the scaling laws applied for cratering we in effect use it as a means of extrapolating on one dimension with two or three other dimensions known.

By modeling is meant that the independent Pi terms of model and prototype are identical, and that therefore, also the dependent Pi terms are equal. Thus, there is no extrapolation.

Example of "scaling" and modeling

The example will presumably be based on experiments done on known systems to predict what will happen to an unknown system.

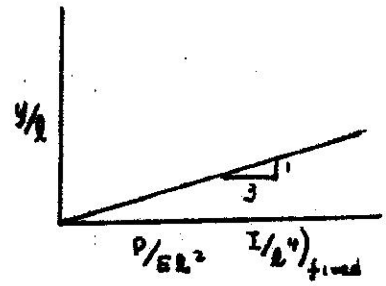
Assume that we want to know the deflection at the end of a given uniform rectangular cantilever beam subjected to a load at the end. We guess that the deflection must be a function of the length of the beam, the size of the load and the moment of inertia of the beam. We may also guess that it is a function of the Young's modulus. Assume that we have several beams and loads available, but of course not one the size of which we want to predict. We might then start by holding all but one of the variables constant, and plot the deflection as a function of that variable. Let us say that we plot deflection vs length and obtain a family of curves.



Now, if the beam for which we want to predict is longer than any of the other beams, it becomes necessary to extrapolate. Even though our curves are "scaled" in the sense that the other parameters may correspond to what we want, extrapolation past a measured point is risky at best, and may be completely erroneous.

If, instead, we use the Buckingham relationship we find that $Y = f(P, \ell, E, I)$ which, expressed in two dimensions gives us three Pi terms. Thus $\frac{Y}{\ell}, \phi\left(\frac{P}{E\ell^2}, \frac{I}{\ell^4}\right)$. A few plots, using no more than 2 beams, would show us that the function would be $\frac{Y}{\ell} = \frac{1}{3}\frac{P}{E\ell^2}\left(\frac{I}{\ell^4}\right)^{-1} = \frac{P\ell^2}{3EI}$

What is more, without finding the formula we are able to locate our unknown beam in the range of the measured quantities, so that it is no longer necessary to extrapolate.



This is the most important point about modeling: That if it is performed correctly it is theoretically possible to predict any phenomenon.

III. Mathematical Equation and Dimensionless Equation

It is obvious that the Pi theorem by itself cannot provide us with a complete solution to any problem. It is necessary to perform experiments to obtain the functional relationships of the Pi terms, as well as provide the necessary numerical constants. However, for those who believe blindly in differential equations, and not at all in dimensional analysis, let us look at the similarities as well as the differences.

Similarities	Differences
1. Same number of independent variables	1. For relatively simple equations with sufficient B. C. the diff. eqn. will yield the solution in a more satisfactory manner.
2. Same functional relationship between variables	2. For very complicated phenomena or those who do not lend themselves to analytic solutions the dimensional analysis and experiment gives us fewer variables and a solution whereas ordinary methods may not.

The two methods are, therefore, complementary rather than competing.

COMMERCIAL RADIOISOTOPE PRODUCTION

M. Saca

Radioisotope production by a nuclear explosion is expected to be much cheaper, and yield much greater quantities, than is presently possible with nuclear reactors. The recovery of isotopes produced in an underground nuclear explosion is a formidable and challenging undertaking. One proposal which shows promise is to use a salt strata, dissolve the salt formation after the shot and then use chemical processes to recover the isotopes produced.

APPENDIX C

HOMEWORK PROBLEMS IN NUCLEAR CIVIL ENGINEERING

As a guide and teaching aid a set of typical problem assignments have been compiled. These problems are keyed to "The Constructive Uses of Nuclear Explosives," by Teller, Talley, Higgins, and Johnson.

(1.1) If one mole of particles is confined to 1 cc and heated to a temperature of "1 electron volt," what will be its pressure, in millions of atmospheres? (Three figure accuracy, please. $kT = 1 \text{ eV} = 1.602 (10^{-12})$ ergs.)

(1.2) In chapter 3, we find that 6 deuterons can combine to yield 2 protons, 2 neutrons, 2 helium-4's, and 43.2 Mev. For devices, the energy yield from the fissioning of a single U^{235} atom is 180 Mev.

(a) On a kilogram-to-kilogram basis, what is the ratio of energy release of complete fission of uranium-235 to the complete fusion of the deuterium in D_2O?

(b) Use the figures of $12,000/kg of U^{235} and $60/kg of D_2O and compare the costs per kilowatt-hour for both fuels.

(1.3) If 95% enriched U^{235} costs $12,000/kgm, what must be the cost of 0.7% enriched U^{235}?

(2.1) A collection of particles, each of mass on, starts at $x = 0$ with $v = 0$ and moves under the influence of a potential $V = -ax$.

(a) Now let the particles have an energy spread from E to $E + \Delta E$, and draw the path of the system in phase space, p versus x.

(b) Consider the phase volume (area) bounded by P_a and P_b at any time, show that this area is the same at any later time t.

(2.2) Consider a system of 3 indistinguishable particles which can occupy 7 different energy levels. The levels are equally spaced and differ by ϵ units -- with the value of the first level being ϵ . If the total energy of the system is 9ϵ ,

(a) What is the average energy per particle?

(b) Prepare a sketch of the energy levels and show the different ways in which the particles can be distributed among them.

(c) Prepare a plot of the frequency of occupation of a level versus the level number. (That is, in one arrangement you will find all three particles in level 3, in another you will find one particle at level 4, one at level 3, one at level 2. You then would say that, so far, level 3 is occupied four times, levels 2 and 4 are each occupied once.)

(d) Compare your plot with the curve $9\ e^{-E/3}$.

(2.3) A strong shock moves up through the ocean floor. Both the earth and the water obey a γ -law. At the hydrostatic pressure of the sea bottom (2×10^{10} dynes/cm^2), the water has a density of 2.5 gm/cc and the earth has a density of 3.0 gm/cc. The water is more compressible ($\gamma_w = 2$) than the earth ($\gamma_e = 3$). If the shock velocity in the rock is 4×10^6 cm/sec, what will be the shock velocity in the water? Assume that the shocked rock unloads as P/ρ = constant.

(2.4) Plot the penetration of the wave

$$\sigma(x) = \begin{cases} 3 \sin x & 0 \leq x \leq \pi/2 \\ 3 \sin (x/2 + \pi/4) & \pi/2 \leq x \leq 3\pi/2 \end{cases}$$

through a vacuum interface that is a distance 2π from the origin. Indicate the points of penetration where the material just goes under tension and where the tension first reaches the maximum.

(2.5) If a saw tooth wave, σ_m and λ , creates exactly 3 spalls, what is the tensile strength of the material? What are the velocities of the slabs?

(2.6) A stress wave moves to the right with a velocity of 500 m/sec. At t = 0, the front of the wave is 1 meter from a vacuum interface. The wave is one meter in length and is of the form

$$(x,t) = \begin{cases} 0 & x < 500\,t \\ 3(x-vt)^2 & 500\,t < x < 500\,t + 1 \\ 0 & 500\,t + 1 < x \\ \text{Undefined for } x > 2 \end{cases}$$

When will a spall occur if the tensile strength of the material is 2?

(2.7) Derive explicitly the relation $\rho_r/\rho_o = \gamma(\gamma + 1)/(\gamma - 1)^2$, for strong shocks reflected off a regid wall.

(2.8) The following table represents the results of solving the Thomas-Fermi equation at absolute zero for hydrogen, ${}_1H^1$.

P (bars)	v x 10^{-22} (cc)	P (bars)	v x 10^{-22} (cc)
3360	0.549	730	1.27
2810	.609	651	1.37
2440	.670	520	1.45
2120	.730	422	1.57
1845	.790	346	1.69
1638	.850	307	1.81
1430	.9111	269	1.94
1181	.971	239	2.06
1100	1.03	212	2.18
1000	1.09	187	2.30
940	1.15	173	2.42
805	1.21		

(a) Use these results to plot a P-v diagram for ${}_{26}Fe^{56}$ between the densities of 12 and 24 gm/cc. ($P_z = Z^{10/3} P_H$, $V_z = Z^{-1} V_H$).

(b) Estimate the work for such a compression at absolute zero.

(c) Is this work greater or less than the real work needed for such a compression?

(2.9) Show that if the potential is of the form V $1/r^n$, the Virial Theorem is

$$mE_{pot} + 2\ E_{kin} = 3\ PV$$

(2.10) Since the Virial Theorem, as derived, holds for inverse square forces, why can we apply the results to the perfect gas?

(3.1) In Chapter 3 we present 8 possible exothermic reactions. The energy releases are given to one decimal place accuracy. Use a chart of the nuclides or the Handbook of Chemistry and Physics, the relation $\Delta E = c^2 \Delta m$, and $c^2 = 931.2$ Mev/AMU to obtain the energy releases correct to <u>two</u> decimal places. (Note that $C^{12} = 12.00386$).

(3.2) Plot, as a function of % of atoms fissioned the cost of a 10 kt explosion if U^{235} is used at a cost of $12,000 per kilogram.

(3.3) Table 3.1 presents the radioactivity due to each radioisotope from a 1 kt fission explosion, as a function of time.

(a) Plot, on log-log paper, the total radioactivity due to this explosion as a function of time.

(b) Find A_1 and n in

$$A(kCi) = A_1\ t^{-n}$$

(3.4) If venting occurs, we might find a region near the site where the fallout is due to the intermediate group (see Table 3.2). Plot the activity expected from these elements. Does the radioactivity decrease faster for this group than for the total yield? Why?

(Hint: $\lim_{t \to \infty} t^n e^{-t} = 0$ for any finite n.)

(3.5) At t = 0 + 1 H, the dose rate near a nuclear explosion is 13.8 R/hr. Construction workers are to go in an work 5 consecutive shifts - 8 hours on, 16 hours off. If their total exposure cannot exceed 30 mr, how many hours after t = 0 can they start the first shift?

(a) What yield should we use?

(b) What will be the cost per cubic yard of rock broken in the chimney?

(4.4) Two detonations are planned, both of 100 kt yield. One is to be performed on the earth and the other on the moon. We assume that the lunar density is 3.34 gm/cc and that the earth has a density of 2.38 gm/cc. The acceleration due to gravity on the moon is 162 cm/sec^2.

(a) If the earth shot is buried 800 m, at what depth should the lunar shot be emplaced to produce the same size cavity?

(b) If the device were buried at 200 m on earth, at what depth should the lunar shot be buried to produce the same size crater? Assume that air drag is negligible on earth so that the two craters are of similar shape.

(4.5) Assume apparent crater is a paraboloid of revolution. Given the scaling law of 1/3.4, and the fact that the Sedan crater (100 kt, DOB 635 ft) had a volume of 6.6×10^6 yds. Assume that a 10 Mt yield has a variation of $\pm$ 10%, that the scaling law varies between 1/3.2 and 1/3.6. What are the upper and lower limits on the volume, the radius, and depth of this larger apparent crater?

(4.6) Overlay three curves of maximum apparent crater volume versus yield, one based on a parabolic relation between depth and radius, another based on a hyperbolic relation, and the third based on a spherical relation. State all assumptions and give the equations or curves you have used as your source material.

(5.1) Develop construction cost estimate curves for placement of nuclear explosives for producing a crater or aquifer on San Clement Island. Assume that rotary drilling is used. List further assumptions that you make and discuss your reasoning.

Reference: TID - 7695

(5.2) Current philosophy in the implacement of nuclear devices requires that the boring be dry. (1965) Determine the wall thickness required of the steel linear 60" inside diameter, 1000 feet below the water table. The lightest weight design is desired.

Reference: Timoshenko, "Theory of Elastic Stability".

(7.1) If one ignores B-decay and assumes that the cross section for neutron capture is constant, show that the equation governing the time-rate-of-change of nuclei with extra neutrons is

$$\frac{d}{dr} N_{\ell} = N_{\ell-1} - N_{\ell} \quad (0) = 0, \quad \ell \geq 1,$$

where the scale time, $r = \sigma \phi t$; is the capture cross-section, ϕ is the flux or neutrons, and t is time.

(7.2) (a) Verify by direct substitution that the solution to this equation is

$$N_{\ell} = \frac{r^{\ell} e^{-r}}{\ell !}$$

(b) At what value of the scale time will the number of nuclei with one extra neutron ($\ell = 1$) be a maximum? At what value of the scale time will the number of nuclei with two extra neutrons be a maximum? ℓ extra neutrons?